

Response behaviour of hot wires and films to flows of different gases

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Measurements of the voltage output for hot-wire and film anemometers placed in flows of nine different gases have been made as a function of flow velocity. It has been possible to correlate the measurements quite accurately by treating the data in terms of suitably defined Reynolds and Nusselt numbers. In order to obtain these correlations it has been necessary to consider and correct for the effects of probe-end conduction losses, temperature dependencies of gas molecular properties, flow slip at the probe surfaces, and gas accommodation. With the exception of the results for helium (for which accommodation effects are strong), the most important correction is shown to be that for the different temperature dependencies of the gas molecular properties. This finding is contrasted with previous studies which have assumed that the largest effect among different gases was due to variations in the Prandtl number. The importance of the nature of the flow over the cylindrical devices to the heat transfer behaviour is described. A previously unreported hysteresis in the heat transfer behaviour for $Re \approx 44$ has been characterized and attributed to the presence or absence of eddy shedding from the heated cylinder.

1. Introduction

The simultaneous measurement of concentration and velocity in turbulent flows of gas mixtures has traditionally been very difficult. Such measurements are necessary before comprehensive models of turbulent mixing in variable-density flows can be developed. Recently, we have demonstrated that the techniques of Rayleigh light scattering for concentration measurement (Pitts & Kashiwagi 1983, 1984) and hot-wire or -film anemometry can be combined in order to make such measurements in isothermal flows (Pitts, McCaffrey & Kashiwagi 1983). This new technique is capable of simultaneous, spatially resolved, real-time measurements of concentration and velocity in turbulent flows of two gases.

In order to apply this diagnostic technique, it is first necessary to calibrate the response of the hot wire or film as functions of concentration and velocity. This may be done by monitoring the response of the heated filament as a function of velocity for several different mixtures of gases covering the concentration range of interest. However, this procedure is tedious and time consuming. A second method would be to have suitable empirical laws so that a calibration of wire response for a single gas would be sufficient to allow the prediction of the wire response for other gases and mixtures.

With the hope of generating such empirical laws we have measured the response of hot wires and films for a variety of gases and binary gas mixtures. In this paper we will describe the behaviour of the wire and film in pure gases. (Note that air is included as a pure gas.) A second paper will describe similar measurements for binary gas mixtures.

Since we are primarily interested in calibrating the anemometers for use in our flow system, we have limited our study to one overheat ratio for the film and one for the wire and to the velocity range of interest in our experiments (approximately 50–1000 cm/s).

As will be shown, we have been able to generate empirical relationships for predicting hot-wire and film response variations as a function of changes in gas composition. In the process we have developed a much better understanding of the behaviour of hot wires and films. Such an understanding is necessary in order to be able to assess the accuracy of velocity measurements in flows of varying composition. These new results also allow a more critical examination of past experiments reported in the literature. In particular, the effects of thermal conduction along the probes, rarefied-gas effects, thermal accommodation on the probe surface, temperature dependence of gas properties, and flow transitions on heat-transfer behaviour are described.

In §2 we will summarize past studies which are relevant to this work. This will be followed by sections describing our experimental techniques (§3) and the results of our measurements (§4). Efforts to develop empirical heat-transfer laws which correlate these measurements will be reported in §5. In §6 we will discuss possible explanations for and implications of our observations concerning the behaviour of the hot wire and film. Finally, §7 contains a summary of important results and conclusions.

A version of this article is available (Pitts & McCaffrey 1985) as a National Bureau of Standards internal report. It contains expanded discussions of previous literature results as well as more complete derivation of some of the heat-transfer equations.

2. Past work

2.1. Results for air

Hot-wire and film anemometry are the most widely used techniques for velocity measurements in fluid flows. Many books and monographs contain extensive discussions of the use and applications of these devices (e.g. Corrsin 1963; Hinze 1975; Bradshaw 1971; Comte-Bellot 1976, 1977; Fingerson & Freymuth 1983; Perry 1982). When such anemometers are employed carefully, they are highly precise and accurate.

As might be expected, the vast majority of gas-velocity measurements using heated-filament anemometers have been made for air. There are very few reports of their use in other gases and still fewer regarding their application in measurements where the gas composition is fluctuating. Here we will describe the relevant findings for air flows and then describe past studies of hot-wire and film responses in different gases.

Most of the hot-wire and film anemometers in use today employ cylindrical conducting wires or conducting films applied to non-conducting substrates. These probes are usually operated in the 'constant-temperature' mode. Since the resistivity of the probe depends on temperature, in practice this means that the resistance of the probe is held constant and the amount of current flowing through the probe is measured to obtain the power dissipation within the probe. If it is assumed that the only heat loss mechanism is due to convection, the power loss of the probe can be related to velocity.

Heat-transfer laws for infinitely long cylinders are often used to predict the behaviour of heated wires and films in gas flows. These laws are usually expressed

in non-dimensional form by giving the Nusselt number $Nu = hD/k$ as a function of Reynolds number $Re = \rho UD/\mu$, where h is the convective heat-transfer coefficient, D the sensor diameter, k the thermal conductivity, μ , the dynamic viscosity, U , the velocity perpendicular to the cylindrical sensor and ρ the density. One of the first heat-transfer laws was due to and bears the name of King (1914). King's derivation was based on the assumption of potential flow, which has been shown to provide a poor approximation of real flows. However, relations based on his work are often used in the form

$$Nu = C_1(Pr) + C_2(Pr) Re^{\frac{1}{2}}, \quad (1)$$

as a basis for empirical heat transfer laws for hot-wires and films. C_1 and C_2 are parameters which depend on Prandtl number $Pr = C_p\mu/k$, where C_p is constant pressure specific heat.

For air or other gases this relation can be rewritten as

$$Nu = A + B Re^n, \quad (2)$$

where the exponent n is now treated as a parameter to accommodate the experimental observation that n may vary. Andrews, Bradley & Hundy (1972) have compiled a list of many of the correlations found in the literature which have been used to predict Nu behaviour as a function of Re .

The non-dimensional correlation that is used most often is due to Collis & Williams (1959). It can be written as

$$Nu_a \left(\frac{T_m}{T_\infty} \right)^a = A_a + B_a Re^n, \quad (3)$$

where the values of the powers a and n , as well as A_a and B_a have the values listed in table 1. These measurements were made for overheat ratios (defined as $(T_s - T_\infty)/T_\infty$, where T_s is the sensor temperature and T_∞ the ambient temperature) in the range of 0.1–1. The fluid properties necessary for (3) are evaluated at the mean temperature of the sensor surface and the ambient gas T_m . A modification of heat-transfer behaviour at $Re \approx 44$ was found. It was abrupt and attributed to the onset of vortex shedding from the cylinder. A similar observation of this flow transition had been reported earlier by Hilpert (1933).

The subscript a in Nu_a refers to a continuum Nusselt number which has also been corrected for accommodation effects. Collis & Williams (1959) showed that rarefied-gas effects are important for small-diameter wires. They derived an approximate correction for these effects, which for air can be written as

$$\frac{1}{Nu_\infty} - \frac{1}{Nu_a} = 2Kn, \quad (4)$$

where Kn is the Knudsen number λ/D where λ is the mean free path of a gas molecule. The subscript ∞ indicates a Nusselt number for a wire of infinite length, i.e. no end losses. These authors also found that buoyancy effects become important in the heat transfer at very low Re . They concluded that buoyancy effects are negligible when

$$Re > Gr^{\frac{1}{4}}, \quad (5)$$

where both Re and the Grashof number $Gr = g(T_s - T_\infty)D^3/\nu^2T_\infty$ (where g is the acceleration due to gravity and ν the kinematic viscosity) are evaluated for air at ambient conditions. For the above overheat ratios, velocities ought to be larger than about 2–5 cm/s to ensure that buoyancy effects can be neglected.

	$0.02 < Re < 44$	$44 < Re < 140$
n	0.45	0.51
A	0.24	0
B	0.56	0.48
a	-0.17	-0.17

TABLE 1. Values found by Collis & Williams (1959) for use in (3)

Bradbury & Castro (1972) verified the results of Collis & Williams (1959) for overheat ratios of 0.07–2.1. These authors reported some temperature-decay measurements for wires heated by current pulses which also supported the findings of Collis & Williams.

Andrews *et al.* (1972) have also investigated the responses of hot wires in air. They have given an excellent discussion of the corrections necessary for rarefied gas and accommodation effects.

Accommodation effects (Kennard 1938) are treated using the accommodation coefficient (α), defined as

$$\alpha = \frac{\epsilon_i - \epsilon_r}{\epsilon_i - \epsilon_s}, \quad (6)$$

where ϵ is the total energy for a molecule and the subscripts i , r and s refer respectively to incident, reflected and fully equilibrated value at the sensor surface temperature. α can be thought of as the ratio of the average increase in energy of the gas molecules after striking the surface to the increase in energy they would have if they remained near the surface long enough to come into equilibrium at the surface temperature. $\alpha = 0$ if the gas 'accommodates' no energy from the surface and $\alpha = 1$ if gas molecules striking the surface come into thermal equilibrium at the surface temperature.

Heat transfer to a rarefied gas from a heated surface is treated using the concept of a temperature jump at the surface (Kennard 1938). For a heated cylinder, the temperature jump is defined as

$$T_s - T'_s = \Delta \frac{\partial T}{\partial r} \quad (7)$$

where T'_s is the temperature the surface would have if the temperature jump did not occur and Δ is the temperature jump distance. Kennard (1938) showed that for a rarefied gas with gas accommodation effects, Δ can be expressed as

$$\Delta = \left(\frac{2-\alpha}{\alpha} \right) \left(\frac{2\gamma}{\gamma+1} \right) \left(\frac{\lambda}{Pr} \right), \quad (8)$$

where $\gamma = C_p/C_v$. A slip parameter which relates Δ to the diameter of the cylinder is often defined as

$$\beta = \frac{\Delta}{D} = \theta' Kn, \quad (9)$$

where $\theta' = \Delta/\lambda$.

Andrews *et al.* (1972) showed that for an ideal gas the Nu for the gas in the absence of rarefied effects and with $\alpha = 1$ could be related to Nu_a by

$$Nu_a = \frac{Nu_\infty}{1 - \phi Kn Nu_\infty}, \quad (10)$$

where

$$\phi = \theta' \left(\frac{2T_s/T_\infty}{1 + T_s/T_\infty} \right)^{0.5+x-y} \quad (11)$$

x and y are the exponents which are found by assuming $\mu \approx T^x$, $k \approx T^y$, and θ' is evaluated at T_s . While not exact, these two expressions do provide a good approximation for the temperature dependencies of these gas properties. For (10) Kn is evaluated at T_m . Equation (11) is derived by assuming T'_s to be nearly equal to T_s . This is generally a valid assumption.

Andrews *et al.* (1972) found that $2.23 < \phi < 2.67$ for air at their experimental conditions. Substituting ϕ in (10) shows that there is rough agreement between their more exact theoretical treatment and the simpler model of Collis & Williams (1959) as given by (4). In fact, for reasons of simplicity, Andrews *et al.* treated their data using (4).

Unlike the work of Collis & Williams (1959), Andrews *et al.* (1972) found that a correction for temperature loading was not required when (4) was used to correct Nu_∞ for rarefied-gas and accommodation effects. For large aspect ratios (active length divided by diameter l/D) they found that the equation

$$Nu_a = 0.34 + 0.65 Re^{0.45} \quad (12)$$

could be used for $0.02 < Re < 20$, $0.02 < (T_s - T_\infty)/T_\infty$ and $0.052 < Kn < 0.1$. Since Collis & Williams (1959) also included Kn -effects in their study it is difficult to understand why the results of Andrews *et al.* (1972) differ from this earlier work. The answer may lie in the different molecular properties used by the two groups (see the discussion in §6.1.3).

The work described above is for heated wires of sufficient aspect ratio that heat transfer from the ends of the wires to the supports can be ignored. In practice, this is often not the case. Andrews *et al.* (1972) found that the aspect ratio was important for $l/D < 400$. They employed an effective Nu_m based on observed wire heat loss and did not correct for end losses. Equation (2) was used to treat their data. They concluded that the value of n remained constant, but that the parameters A_m and B_m increased as l/D decreased. These observations are consistent with earlier experiments reported by Collis (1956) and Champagne & Lundberg (1966). The increase of B_m was shown to be much less rapid with decreases in l/D than was the A_m value. This is not unexpected since the A_m value is generally associated with free convection and conduction losses while B_m is believed to be primarily dependent on forced convection.

Several authors (e.g. Corrsin 1963; Hinze 1975; Fingerson & Freymuth 1983; Perry 1982; Lowell 1950; Davies & Fisher 1964) have discussed the effects of probe-end conduction on measured heat loss and have provided methods for calculating the predicted heat loss in the absence of conduction. Most of these derivations are based on a simplified heat-transfer differential equation which can be written in terms of the temperature difference $\theta = T - T_\infty$ as

$$\frac{d^2\theta}{dx^2} - C_0\theta + D_0 = 0, \quad (13)$$

where

$$C_0 = \frac{\pi D h}{A_s k_s} - \frac{I^2 \rho_r(0) \alpha_p}{A_s k_s} \quad (14)$$

and

$$D_0 = \frac{I^2 \rho_r(0)}{A_s k_s}. \quad (15)$$

In (14) and (15) I is the current through the anemometer circuit probe leg, $\rho_r(\theta)$ the probe resistance per unit length and α_ρ the thermal coefficient of probe electrical resistance. Clearly D_0 is > 0 . For practical flows, C_0 is also found to be greater than zero.

The above expressions assume no radial temperature gradients, no temperature dependence of probe thermal conductivity k_s , constant probe diameter, and area A_s and negligible radiative heat losses.

In solving (13) for a heated probe oriented perpendicularly to the flow it is usually assumed that the temperature distribution is symmetrical about the centre of the probe ($x = 0$), which requires that $d\theta/dx = 0$ at $x = 0$. The probe is also assumed to be attached to massive prongs having an infinite heat capacity and a temperature equal to the ambient flow temperature. This condition can be written as $\theta = 0$ at $x = \pm \frac{1}{2}l$. The solution of (13) for these boundary conditions is

$$\frac{\theta}{\theta_0} = 1 - \frac{\cosh(C_0^{\frac{1}{2}} x)}{\cosh(\frac{1}{2}l C_0^{\frac{1}{2}})}, \quad (16)$$

where $\theta_0 = D_0/C_0$ is the overheat which would exist in the absence of end heat losses due to conduction.

By integrating along the probe, an expression can be obtained for the average overheat $\bar{\theta}$,

$$\frac{\bar{\theta}}{\theta_0} = 1 - \frac{2}{l C_0^{\frac{1}{2}}} \tanh(\frac{1}{2}l C_0^{\frac{1}{2}}). \quad (17)$$

In the hot-wire literature the effects of end conduction are often discussed in terms of the 'cold length' l_c , introduced originally by Betchov (1984). The cold length is related to the above equations by $l_c = C_0^{-\frac{1}{2}}$ or $1/C_0^{\frac{1}{2}}$.

The above description of probe thermal-conduction effects is related most closely to the development given by Corrsin (1963). We will use these results in §3.7 to develop a method for correcting our experimental results for end-conduction losses.

Collis & Williams (1959) found that for large-aspect-ratio wires it was necessary to multiply Nu_a by $(T_m/T_\infty)^{-0.17}$ to correlate their results (see (3)). Koch & Gartshore (1972) reported a study for 5 μm wires having aspect ratios of 230. Even though these workers did not correct their results for end conduction losses or accommodation effects, they fit their data to an equation of the same form as (3). Nusselt numbers based only on experimentally observed heat losses are denoted Nu_m . In agreement with Collis & Williams (1959) it was found that plots of Nu_m versus Re were fitted best for $n = 0.45$. As expected, based on the discussion above, their measured values of $A_m = 0.72$ and $B_m = 0.80$ are larger than found by Collis & Williams (see table 1) and A_m is increased proportionally more than B_m . The major surprise of this work is the measured value of $a = 0.67$. This value of a has a different sign and much larger magnitude than found by Collis & Williams (1959). This difference was attributed by Koch & Gartshore (1972) to aspect-ratio effects.

Bruun (1975) also reported a study of the effect of overheat ratio on the response of hot wires when operated in the constant-temperature mode. The hot wires used in this study had aspect ratios varying from 360 to 420. The results of this study were not treated in terms of (3), but rather in terms of the observed voltage drop across the hot wire E_s ,

$$\frac{E_s^2}{R_s(T_s - T_\infty)} = A' + B' U^n, \quad (18)$$

which is proportional to Nu_m . Here R_s is the electrical resistance of the sensor.

Bruun found that a value of $n = 0.45$ fitted his results well for velocity ranges corresponding to $Re < 44$. Even though his measurements were not analysed in the form of (3), he was able to show that the value of a in this equation depended quite strongly on velocity. His measured values of a differed markedly from those of Collis & Williams (1959) and Koch & Gartshore (1972).

In treating his data, Bruun mentioned potential problems in obtaining accurate values of E_s which can arise when utilizing standard constant-temperature-anemometry circuitry. Such circuits consist of a Wheatstone bridge, one leg of which is formed by the probe, and a feedback amplifier which responds to variations in probe resistance in such a way as to maintain a constant resistance. Usually, these amplifiers also include a small offset voltage which is designed to force the bridge slightly out of balance in order to obtain a minimum damping of the amplifier output. In this manner the frequency response of the device is maximized. Owing to this offset voltage, a measurement of the voltage drop across the bridge does not provide a true measure of the voltage drop across the probe. Bruun overcame this problem by measuring the voltage drop across different legs of the bridge.

The effects of offset voltage on constant-temperature anemometry was discussed in much greater detail by Morrison (1976). He provided a theoretical analysis which showed that the offset-voltage setting can modify the observed voltage drop across the bridge and lead to what appears to be a probe overheat dependence which is in reality an artifact due to the electronics. Experiments were reported which supported the theoretical conclusions. Morrison notes how this artifact can result in errors in measured heat-transfer laws and proposed this effect as the source of disagreement between the measured value of a for the work of Collis & Williams (1959) and Koch & Gartshore (1972).

It seems clear from these papers that differences in observed heat-transfer laws for hot-wire anemometers should be expected unless great care is taken to account for end conduction losses and response behaviours of the electronics used.

2.2. Results in different gases

Most of the relevant studies on hot-wire behaviour in different gases have resulted from efforts to use two hot wires (and/or films) to make simultaneous concentration and velocity measurements. As first noted by Corrsin (1949), such measurements are possible when the heat loss of the two heated filaments is different for a given velocity and concentration. By simultaneously monitoring the heat loss of each wire, the concentration and velocity can be determined from suitable calibration curves.

Even though there had been previous efforts to use the ideas of Corrsin (1949) to extend hot-wire anemometry to variable-composition flows (Conger 1965; Baid 1967; Tombach 1969) the first literature reports appear to have been due to Kassoy (1967) and Aihara, Kassoy & Libby (1967) who investigated the response of hot wires in mixtures of helium and nitrogen as a first step in calibrating hot wires for making simultaneous concentration and velocity measurements. They attempted to use a modified King's law to predict the response of the wire when placed in different gases. By using measurements in nitrogen as a reference the voltage required by the hot wire to maintain a constant temperature in flows of helium and mixtures of nitrogen and helium for $Re \approx 10$ was predicted. To their surprise, they found that the calculated heat loss, which is proportional to E^2 , was much less than predicted. For this reason, they performed an extensive theoretical (Kassoy 1967) and experimental (Aihara *et al.* 1967) study for $Re < 0.1$. These studies indicated that the observed discrepancies were due to temperature-slip effects at the surface of the wire. Even though the Kn were in a range (~ 0.01) where such effects are generally considered

to be small, the extremely low thermal accommodation coefficient (α) of helium on tungsten greatly increases the importance of slip effects. A theory was developed by Kassoy (1967) which predicted the behaviour of Nu_x in terms of the flow parameters, gas properties, and the slip parameter (β) as defined by (9).

The above studies were performed for an Re range which is much lower than generally used in hot-wire studies. Furthermore, the theoretical study (Kassoy 1967) considered infinite cylinders and the experimental work (Aihara *et al.* 1967) used wires having large l/D ratios and guard heaters to minimize heat transfer to the prongs. For these reasons, this early work is not directly applicable to the hot wires generally used in practice. However, they do contain several findings that are necessary to understand the response of practical hot wires and films in variable-composition flows. As already discussed, the importance of thermal-slip effects was noted. These effects were shown to be much less important for nitrogen than for helium. In fact, the heat transfer of the tungsten wire in nitrogen could be predicted without including the effects of thermal slip as shown by the agreement between the predictions of Kassoy's (1967) theory and experiment. Both the measurements and theory were in excellent agreement with the empirical correlation of Collis & Williams (1959). These papers emphasized the problems in predicting the effects of thermal accommodation behaviour and the lack of theories for predicting α in mixtures.

A third paper from this same group reported measurements similar to those discussed above for nitrogen-neon mixtures as well as further measurements of nitrogen-helium mixtures (Baccaglioni, Kassoy & Libby 1969). It was shown that the thermal accommodation coefficient of neon on tungsten must be considered in using Kassoy's (1967) theory to predict the Nu_∞ for a given flow velocity. A simple gas-kinetic-theory argument was used to predict the behaviour of α as a function of neon concentration.

Wu & Libby (1971) later reported Nu_∞ measurements versus Re for an Re range of 0.03–10 for air, helium, and mixtures of the two gases. In this way they were able to compare the results at lower Re with earlier work from the same group (Kassoy 1967; Aihara *et al.* 1967; Baccaglioni *et al.* 1969) and at the same time obtain measurements over an Re -range of more interest for practical hot-wire measurements. These experiments were done using a platinum wire instead of tungsten. In analysing their data these workers introduced a modified version of (3) using the values of A , B , and n listed in table 1 for $Re < 44$,

$$Nu_\infty = \left(\frac{T_m}{T_\infty}\right)^{0.17} \left[0.24 \left(\frac{Pr_x}{Pr_{\text{air}}}\right)^{0.20} + 0.56 \left(\frac{Pr_x}{Pr_{\text{air}}}\right)^{0.33} Re^{0.45} \right], \quad (19)$$

where x refers to the gas under investigation. The Pr dependence in (19) is taken from the work of Kramers (1946). Kramers deduced his correlation for heated cylinders, which differs slightly from that of Collis & Williams (1959), by using data for air, water, and several viscous oils. It is somewhat alarming that in deducing this relation he dismissed the existence of a flow transition at $Re \approx 50$. This is the same flow transition that is clearly shown in the work of Collis & Williams (1959).

Wu & Libby (1971) found measurements of Nu_∞ as a function of Re for air to be in excellent agreement with predictions using Collis & Williams' relationship (3). However, when they investigated pure helium and mixtures of helium and air, the observed Nu_∞ for a given Re was lower than predicted by (19). Differences between the predictions and experiment were attributed to thermal slip. These workers used

the parameter β' to characterize the effects of thermal slip by an equation which is very similar to (4),

$$\frac{1}{Nu_\infty} - \frac{1}{Nu_a} = \frac{1}{2}\beta', \quad (20)$$

where β' is treated here as an empirical parameter. Based on experimental results, they estimated β' to be 0.274 which gives $\alpha = 0.11$ from (8) and (9). An empirical relationship that assumed that β' for helium-air mixtures was a linear function of helium mole fraction gave limited agreement with experimental results.

Several additional studies of the effects of gas variations on hot-wire response are found in the literature (Wasan, Davis & Wilke 1968; Wasan & Baid 1971; Andrews *et al.* 1972; McQuaid & Wright 1973). These are generally not extensive enough to allow the effects of changes in gas composition on hot-wire behaviour to be fully characterized.

Simpson & Wyatt (1973) have investigated the heat loss of various hot films in air, helium and argon and in helium-air and argon-air mixtures. Both cylindrical and parabolic sensors were considered. These authors used the modified Collis & Williams' heat-transfer law given in (19) as a basis for analysis of their results. Since cylindrical hot films have very small aspect ratios, it was necessary to allow the numerical constants (0.24 and 0.56) in (19) to be parameters and determine them experimentally. Using the resulting equation, they were able to obtain an excellent correlation of their experimental results for argon, air and mixtures of the two.

However, when they considered helium and helium-air mixtures they found that the modified Collis & Williams' equation overpredicted the convective heat transfer from the film to the gas. This observation is similar to those found by Libby and coworkers (Aihara *et al.* 1967; Baccaglioni *et al.* 1969; Wu & Libby 1971) for helium-air and helium-nitrogen mixtures. In this earlier work, the decrease in Nu_∞ from that expected was attributed to thermal accommodation effects on the surface of the platinum and tungsten wires. However, Simpson & Wyatt (1973) noted that α for helium on quartz (the overcoating used on the film sensor) is expected to be larger than on the metals used for construction of the wires. The result is that thermal accommodation effects should be much less important for films than for wires. For this reason, these authors attribute the decrease in Nu_m to thermal-diffusion effects. There is apparently no theory which allows the prediction of such effects on hot-wire behaviour. It should be noted that Khalifa, Kestin & Wakeham (1979) have considered the effects of thermal diffusion in the context of hot-wire thermal-conductivity measurements. We shall discuss the effects of thermal diffusion and develop an approximate theory to describe these effects in a subsequent paper dealing with hot-wire behaviour in mixtures.

Some work has been reported on the heat transfer from hot surrounding gases to cooled films (Fingerson & Ahmed 1976). These experiments are obviously related to the hot-wire studies, but the authors argue that the same heat-transfer laws should not hold because of differences in the dynamical dissimilarity with temperature loading between heating and cooling of rods. Measurements were reported in heated flows of N_2 , He, CO_2 , and various mixtures of these gases. Their studies covered a range of ambient temperatures from 800 to 1600 K and film-surface temperatures from 350 to 525 K. For $5 < Re < 44$ the results could be well correlated by

$$Nu_m = (0.21 + 0.50 Re^{0.45}) \left(\frac{\nu_\infty}{\nu_m} \right)^{-0.15}. \quad (21)$$

This equation is clearly similar to (3) when the parameters of table 1 for $Re < 44$ are used. However, the temperature-loading factors lead to different calculated Nu based on the two expressions.

It is interesting that Fingerson & Ahmed (1976) have been able to correlate their measurements of He, CO₂, N₂, and mixtures of these gases. This implies that for their experimental conditions such effects as thermal slip and thermal diffusion are absent. This observation is consistent with the large Kn for the film and the large accommodation coefficient for helium on quartz as pointed out by Simpson & Wyatt (1973).

Brown & Rebollo (1972) have given a general discussion of the response of a hot wire to variations in the concentration of a constant-velocity flow in connection with their development of an analytical probe for concentration measurements in binary gas mixtures. As a part of this study these workers attempted to predict the wire response to different gases using heat-transfer laws similar to those previously discussed. For a platinum wire they concluded that it is necessary to consider accommodation effects when helium and argon are used. Tombach (1969, 1973) has also briefly described the importance of thermal-slip effects on hot-wire measurements and noted a memory effect for hot wires placed in helium and then used to measure air flow velocities.

3. Experimental system and data treatment

3.1. Flow system

The response of the hot wire and film were investigated as a function of velocity using a TSI model 1125 flow calibrator†. This device consists of a flow regulator, baffles and flow straighteners, and carefully machined nozzles of decreasing size which are designed to provide uniform velocity profiles. The velocity probe is aligned perpendicularly to the flow direction at a small distance above the exit of the nozzle.

We have investigated flows of relatively low velocity (approximately 50 cm/s–1000 cm/s). For these velocities the gases can be considered to be incompressible to a very high degree of accuracy. In this case, the pressure drop ΔP across the nozzle is related to velocity by the Bernoulli equation.

The difference in pressure on either side of the nozzle was determined using a Datametrics electronic manometer (type 1014A) equipped with a 1 Torr differential head which could be used for pressure measurements to 1.3 Torr. The pressure reading of the instrument is available as a voltage which was read by a 3½ digit digital volt meter. The performance of the electronic manometer was checked against a water column micromanometer.

3.2. Gases

Experiments have been performed for a wide variety of gases obtained from many suppliers. Only for methane (98%) were stated purities less than 99%.

3.3. Gas properties

A knowledge of their physical properties as a function of temperature is required for the gases used in this study. Densities have been obtained from known values at

† Certain commercial equipment, instruments, or materials are identified in this paper in order to adequately specify the experimental procedure. Such identification does not imply recommendation or endorsement by the National Bureau of Standards, nor does it imply the materials or equipment are necessarily the best available for the purpose.

given temperatures and extrapolated to the temperature of interest, assuming ideal gas behaviour. In recent years reliable values of μ , k and C_p have become available for most of the gases we have investigated. Where possible, we have chosen to use the values listed in the Thermophysical Properties of Matter, The TPRC Data Series (Touloukian, Liley & Saxena 1970; Touloukian, Saxena & Hesterman 1975; Touloukian & Makitas 1970) which is published by the Thermophysical Property Research Center of Purdue University. The only exception is the thermal conductivity of propane which is taken from a more recent measurement (Ehya, Faubert & Springer 1972). The values found in these tables are based on extensive reviews of the existing literature (up to the early 1970s) and critical analysis of these results. Uncertainties are quoted for the values over various temperature ranges.

Physical properties for gases not listed in the TPRC Data Series are taken from various sources. Table 2 lists references to property data for all the gases studied here.

During the course of this work it was necessary to calculate mean free paths for some of the gases investigated. We have chosen to do this using the relationship between viscosity and the mean free path:

$$\lambda = \frac{2\mu}{\rho\bar{c}}, \quad (22)$$

where \bar{c} is the average velocity of the molecule, which for an ideal gas is equal to

$$\bar{c} = \left(\frac{8RT}{\pi M}\right)^{\frac{1}{2}}, \quad (23)$$

where R is the gas constant and M the molecular weight of the gas. This same relation was used by Collis & Williams (1959) in their classic study.

3.4. Anemometer electronics

A standard commercial anemometer system was used in this study. This system consists of several modules. A TSI model 1051-2 monitor and power supply generates the necessary voltages for the anemometer system and also outputs a voltage equal to that necessary to maintain the sensor bridge circuit in a balanced condition. This voltage is measured with a 4 digit voltmeter having a 0.1 s time constant.

The constant resistance of the sensors is maintained by a balanced 5:1 Wheatstone bridge. This bridge is contained in a TSI model 1054B linearized anemometer module. Note that the linearizing circuits were not utilized in this investigation and that the recorded output is the bridge voltage necessary to maintain the balance of the bridge. We have not made corrections for the voltage offset of the high-gain amplifier used in the feedback circuit to maintain a constant resistance. This value was set as recommended in the operating manual for the anemometer. Checks showed that the measured bridge voltage was very insensitive to the setting of the offset voltage for the present situation of a single temperature setting.

The resistance of the probe leg of the bridge is determined by the external application of a known control resistance to the model 1054B. A variable decade resistor (TSI model 1056) is used for this purpose. The actual resistance of the heated filament (R_s in table 3) is obtained by subtracting the internal probe and connecting cable resistances from the total resistance of the sensor leg of the bridge (R'_s).

The power dissipated in the wire or film is calculated from a knowledge of the resistance and current flow of the heated filament using $P_s = i^2 R_s$. The current through the sensor is equal to V/R_T where R_T is the total of the standard resistance

gas	k	μ	C_p
Air	Touloukian <i>et al.</i> (1970)	Touloukian <i>et al.</i> (1975)	Touloukian & Makitas (1970)
N ₂	Touloukian <i>et al.</i> (1970)	Touloukian <i>et al.</i> (1975)	Touloukian & Makitas (1970)
He	Touloukian <i>et al.</i> (1970)	Touloukian <i>et al.</i> (1975)	Touloukian & Makitas (1970)
CH ₄	Touloukian <i>et al.</i> (1970)	Touloukian <i>et al.</i> (1975)	Touloukian & Makitas (1970)
Ar	Touloukian <i>et al.</i> (1970)	Touloukian <i>et al.</i> (1975)	Touloukian & Makitas (1970)
CO ₂	Touloukian <i>et al.</i> (1970)	Touloukian <i>et al.</i> (1975)	Touloukian & Makitas (1970)
C ₃ H ₈	Ehya <i>et al.</i> (1972)	Touloukian <i>et al.</i> (1975)	Touloukian & Makitas (1970)
CF ₄	ASHRAE (1976)	Hellmans <i>et al.</i> (1973)	ASHRAE (1976)
CF ₃ Br	ASHRAE (1976)	ASHRAE (1976)	ASHRAE (1976)
SF ₆	Bakulin & Ulybin (1978)	Hellmans <i>et al.</i> (1973)	Adler & Yaws (1975)

TABLE 2. Sources for the properties of gases used in this investigation

	$D(\text{cm})$	$l(\text{cm})$	$R_s(\Omega)$	$T_s(\text{K})$	$T_m(\text{K})$	$\alpha_p(\text{K}^{-1})$	$k_s(\text{cal/s cm}^2 \text{ K})$
Wire	0.0004	0.125	12.02	569	431	0.00363	0.43
Film	0.0051	0.10	9.60	650	472	0.00232	0.0075

TABLE 3. Hot-wire and -film properties used for calculations

in series with the probe leg (R_0) and R'_s . These resistance values are all available and the total power dissipation within the heated filament can be written as

$$P_s = \frac{E^2 R_s}{(R'_s + R_0)^2}. \quad (24)$$

3.5. Probes

All of the measurements reported in this work were made for one wire and one film. The wire measurements employed a TSI 1210-T1.5 tungsten wire which is coated with a very thin layer of platinum to minimize surface oxidation. Table 3 summarizes the dimensional data, operating resistance, the operating temperature of the wire T_s assuming a uniform temperature along the entire length of the wire, overheat ratio $(T_s - T_\infty)/T_\infty$, and T_m defined as the average of T_∞ and T_s . The active length of the wire is determined by gold plating placed on the prong ends of the wire. These wire parameters have been calculated using data supplied by TSI. The operating temperature is calculated using the relation.

$$\theta = \frac{1}{\alpha_p} \frac{R_s - R_\infty}{R_\infty}, \quad (25)$$

where R_s and R_∞ are the resistances of the wire at the operating and ambient gas temperatures respectively. α_p is calculated from TSI calibrations of the wire resistance at the ice and steam points of water.

The film element is a TSI 1205-20. These probes are manufactured by depositing a thin film of platinum onto a cylindrical quartz substrate. The active length of the sensor is then defined by plating a layer of gold (about 6.4 μm thick) at both ends that are attached to the probe mount. The layer of gold provides a low resistance path to the platinum as well as a means for holding the ends of the active area at a temperature very nearly equal to that of the prongs (T_∞). Finally, the entire probe is overcoated with a protective layer of alumina. The exact thicknesses of the substrate and coatings of these devices are difficult to measure and are expected to vary from probe to probe. After consultation with TSI, we have assumed the following dimensions for our calculations: substrate diameter = 50 μm , thickness of platinum $t_{\text{Pt}} = 0.1 \mu\text{m}$ and thickness of alumina $t_{\text{Al}} = 0.8 \mu\text{m}$. The film parameters listed in table 3 are obtained in the same manner as those for the wire.

For the conditions utilized in this work, radiation losses of both the hot wire and film were negligible.

3.6. Experimental procedure

The following experimental procedure was used for measuring the responses of the hot wire and film to varying velocities of the different gases. A cylinder of the pure gas was connected to the calibrator. The gas was allowed to flow for a period of time sufficient to flush the entire flow system. The electronic manometer was then zeroed for a zero flow velocity. Pairs of measurements of pressure drop across the nozzle ΔP

and voltage drop across the bridge E were then recorded for velocities spanning the range of interest. Generally, 20–35 measurements were taken. The readings were taken from high to low velocities and back to high velocity to ensure the absence of systematic or time-dependent effects. The zero-velocity calibration of the manometer was checked often.

3.7. Data analysis

The results of our measurements on the hot wire and film have been analysed in several different ways. First the data are fitted using the expression

$$E^2 = A' + B'U^n. \quad (26)$$

The value of E used is the experimentally observed voltage drop across the bridge and the velocity U is calculated from the observed pressure drop.

The exponent n is treated as a variable. Linear least-squares fits of the data are obtained using a range of different n -values (steps of 0.01). The goodness of fit for these lines can be quantified by using the parameter

$$\chi^2 = \Sigma(E_m^2 - E_{com}^2)^2, \quad (27)$$

where the subscripts m and com refer to measured and computed values of E^2 . For a given n the linear least-squares fitting procedure generates a minimum value of χ^2 . χ^2 varies as a function of n and we have defined the optimum value of n (denoted as n_0) as that which gives the smallest χ^2 .

A crude estimate of the uncertainty in a single point for the fitting of E^2 is obtained by taking the square root of χ^2 and dividing by the total number of measurements. The relative error is estimated by dividing this value by the median value of E^2 .

The measured Nusselt and Reynolds numbers are then calculated for each point and the results fitted to (2). Nu_m is calculated as

$$Nu_m = \frac{P_s}{\pi k l (T_s - T_\infty)}, \quad (28)$$

where P_s is given by (24). All of the properties of the gas under study used to calculate Nu_m and Re are taken at the appropriate T_m listed in table 3 for the wire or film. Note that for any one gas, Nu_m is related to E^2 and Re is related to U by constants. For this reason, the n_0 for use in (2) and (26) are identical.

We have also corrected Nu_m for end-conduction effects to obtain Nu_∞ . Equations (14), (15) and (17) are used for this purpose. Equation (17) can be rewritten as

$$\frac{\bar{\theta}}{D_0} = \frac{1}{C_0} - \frac{2 \tanh(\frac{1}{2}lC_0^{\frac{1}{2}})}{lC_0^{\frac{3}{2}}}. \quad (29)$$

Equation (15) shows that D_0 is a function of I , $\rho_r(0)$, A_s and k_s . All of these values can be measured or estimated for each experimental data point and a value of D_0 calculated. The values of k_s used for the hot wire and film are included in table 3. For the wire this is a literature value (Touloukian *et al.* 1970*a*) of tungsten at 293 K. The hot-film value is obtained by summing values of $A_{qt}k_{qt}$, $A_{Pt}k_{Pt}$, and $A_{Al}k_{Al}$ (where qt refers to quartz, Pt to platinum and Al to alumina) and dividing by A_s to give a k_{eff} for the probe. Necessary values for thermal conductivities are taken from (Touloukian *et al.* 1970*a, b*). This procedure must be considered a crude approximation since it ignores the effects of radial temperature gradients on the heat-transfer process.

Since the value of $\bar{\theta} = T_s - T_\infty$ is also known, (29) can be solved for a value of C_0 .

In practice, this is done by comparing a table of C_0 values as a function of $\bar{\theta}/D_0$ with the $\bar{\theta}/D_0$ value determined using T_s from table 3 and D_0 calculated from (15).

Once a value of C_0 is obtained, it is substituted in (14) along with values of A_s , k_s , I^2 , $\rho_r(0)$ and α_ρ to yield a value for πDh where h is the heat transfer coefficient for the wire or film when end losses are zero. Nu_∞ is obtained from πDh by dividing by π and k_m for the gas.

Once values of Nu_∞ and Re are available for the entire data set, a least-squares analysis identical with that described above is used to fit the results to the form of (2).

Values of Nu_∞ are then corrected for thermal-slip effects. This is first done assuming that the thermal accommodation of the gas on the probe surface is perfect (i.e. $\alpha = 1$). With this assumption, values of Nu_c are calculated from Nu_∞ using (10) with ϕ given by (11) and θ'_{T_s} calculated using (8) and (9). The results of these calculations are then fitted to (2) and values of n_0 , A_c and B_c are determined. For gases for which it is necessary to consider accommodation effects, the above calculations are repeated using a suitable value of α in (8). The Nu which results from this calculation is denoted Nu_a and the parameters which result from fits to (2) are denoted as n_0 , A_a and B_a .

The above calculations have been programmed on a minicomputer. In order to obtain values of A , B and n it is only necessary to input experimental values of E and ΔP along with the physical properties of the gas under study.

4. Results

4.1. Hot wire

Figure 1 shows a plot of E versus U for the response of the hot wire in a flow of air. This data has been fitted to (26) using the procedure described in §3.7. Equation (26) was found to fit the data quite well. A plot of χ^2 (defined by (27)) versus n is shown in figure 2. It is clear from this figure that the value of n which gives the best fit of (26) (denoted by n_0) lies between 0.42 and 0.43. The sharpness of the minimum in the plot of χ^2 versus n has been found to depend strongly on the quality of the experimental data. Smaller uncertainties in the data result in much more clearly defined values of n_0 . The results shown in figure 2 are typical of those that have been found for all of the measurements using the hot wire.

For the data shown in figure 1 we find that the best parameters for use in (26) are $n_0 = 0.43$, $A' = 3.618 \pm 0.0004 \text{ V}^2$, and $B' = 0.3649 \pm 0.0003 \text{ V}^2 (\text{s/cm})^{0.43}$. The error limits given refer solely to those arising from the impreciseness of the linear least-squares fit to the data and are not intended to be an estimate of the actual experimental uncertainty in the parameters A' and B' . The A' , B' , and n_0 values have been used to generate the solid line shown in figure 1, which is in excellent agreement with the experimental results. Using the definition given in §3.7, the relative uncertainty for a single point compared with the calculated line is estimated to be 0.0016. An uncertainty of this magnitude is consistent with the numbers of significant figures available in the measurements of E and U .

The response of the hot wire to changes in velocity has been calibrated for ten different pure gases. These gases include nitrogen and air. The results for nitrogen and air were so similar that we have only reported the results for air. Figure 3 shows examples of E^2 versus $U^{0.43}$ for eight of these gases. The strong dependence of the hot-wire heat loss at a given velocity for different gases is clear from this figure. The results for helium are not included here since these measurements were made over

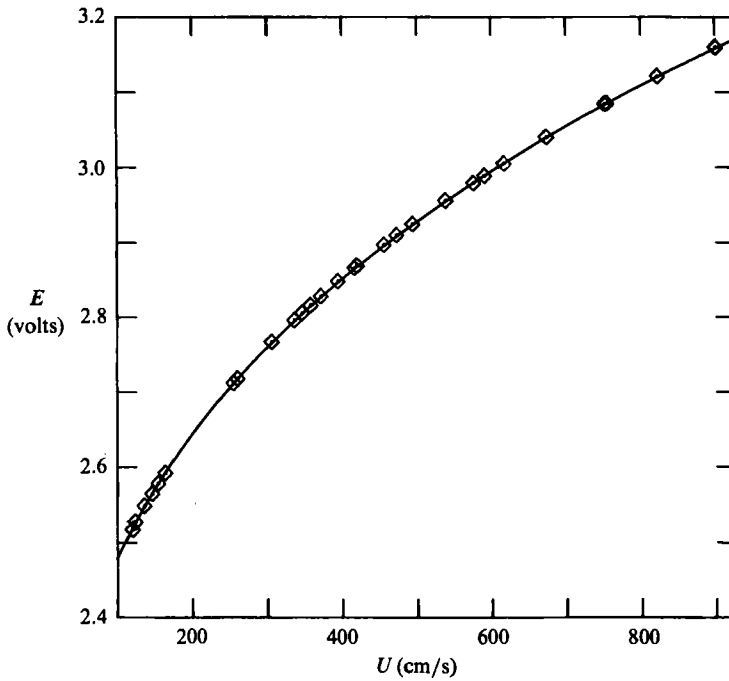


FIGURE 1. The response of the hot-wire to varying velocities of an air flow is plotted as E (volts) versus U (cm/s). The solid curve is that predicted based on a least-squares curve fit of the experimental data to (26). A value of $n = 0.43$ has been used.

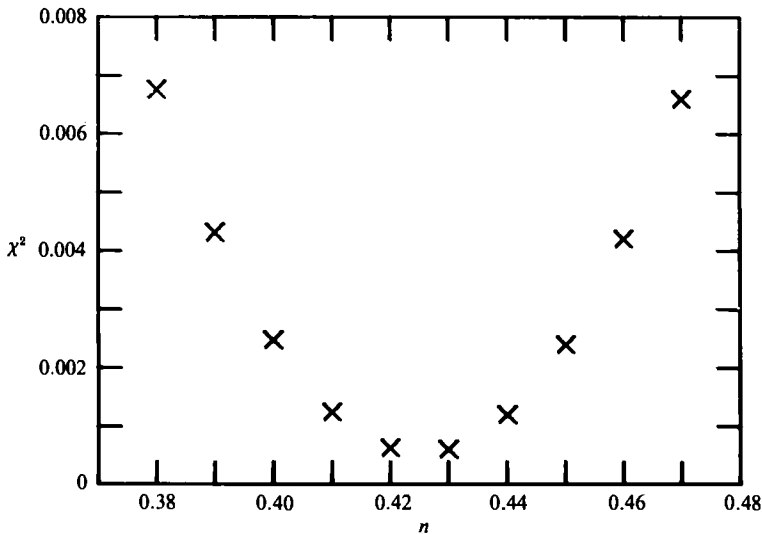


FIGURE 2. Values of χ^2 (defined by (27)) are plotted as a function of the value of n used to fit the experimental results of figure 1 to (26).

a wider range of velocity and the bridge voltage is considerably larger than for the other gases.

Table 4 summarizes the results for all of the hot-wire calibration runs that have been made. Values of n_0 are listed in the first column. These measurements show that all of the experimental data, with the exception of that for helium, can be fitted

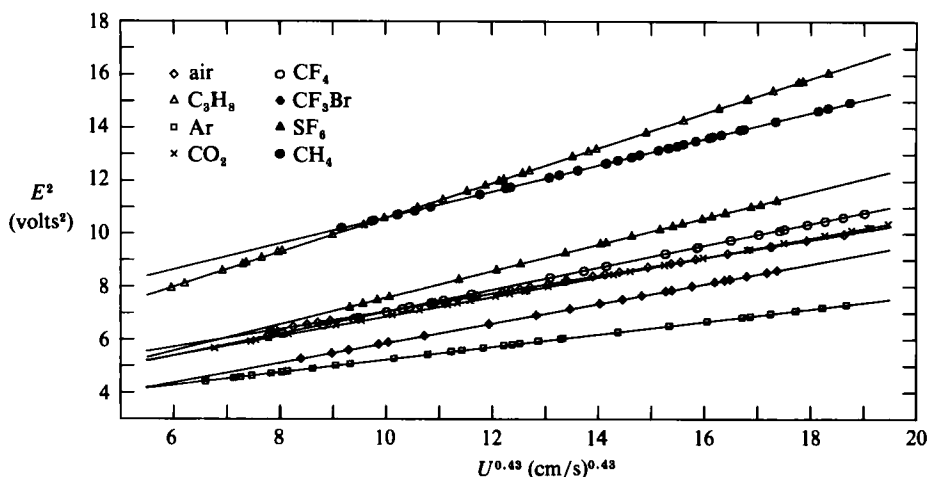


FIGURE 3. Representative data are shown for the response of the hot wire to flows of eight different gases. The experimental results are plotted as E^2 (volts²) versus $U^{0.43}$ (cm/s)^{0.43}. Solid lines correspond to least-squares curve fits of the data to (26) with $n = 0.43$.

Gas	n_0	A'	B'	Re -range	A_m	B_m	A_∞	B_∞
Air	0.43	3.749	0.355	0.09-1.4	0.427	0.664	0.258	0.617
	0.42	3.716	0.340	0.12-1.3				
	0.43	3.716	0.336	0.16-1.2				
	0.45	3.768	0.339	0.23-1.2				
	0.43	3.742	0.339	0.18-1.2				
	0.43	3.723	0.339	0.19-1.2				
CH ₄	0.44	5.667	0.494	0.21-1.1	0.423	0.663	0.301	0.624
	0.43	5.667	0.488	0.17-1.1				
	0.43	5.651	0.497	0.19-1.1				
Ar	0.41	2.852	0.240	0.12-1.4	0.487	0.671	0.250	0.622
	0.42	2.865	0.241	0.12-1.4				
C ₃ H ₈	0.44	4.058	0.658	1.28-1.8	0.492	0.825	0.292	0.743
	0.43	4.051	0.658	1.27-1.8				
	0.43	4.051	0.655	1.27-1.8				
CO ₂	0.42	3.180	0.370	0.21-2.4	0.478	0.741	0.263	0.691
	0.43	3.167	0.368	0.23-2.2				
	0.43	3.206	0.369	0.24-2.2				
	0.44	3.184	0.366	0.27-2.2				
	0.44	3.200	0.365	0.26-2.2				
He	0.26	14.92	0.431	0.18-0.7	0.323	0.372	0.275	0.356
	0.29	15.06	0.423	0.10-0.8				
SF ₆	0.42	2.575	0.517	1.3-5.5	0.489	0.765	0.229	0.717
	0.41	2.656	0.519	1.6-6.1				
CF ₃ Br	0.42	2.212	0.382	0.94-5.1	0.538	0.744	0.213	0.719
	0.42	2.192	0.380	1.3-6.2				
CF ₄	0.42	2.987	0.431	0.50-4.0	0.470	0.711	0.246	0.665

TABLE 4. Values of n_0 resulting from fits of $E^2 = A' + B'U^n$ are listed for the hot-wire measurements made with different gases. Values of A' and B' are reported for $n = 0.43$. Also included are selected slopes and intercepts resulting from least-square fits of Nu_m and Nu_a to (2) with $n = 0.43$. These sets of data correspond to those seen in the various figures. The Re -ranges over which the various measurements were made are also listed

accurately to (26) using $n = 0.43$. For this reason, the values of A' and B' reported in table 4 are fits of (26) with $n = 0.43$.

The measurement results summarized in table 4 were obtained over a period of several months. In general, both A' and B' values for a given gas were found to be reproducible to within approximately 0.5%. While small, these variations are considerably larger than the uncertainty in the fit of a set of data recorded during a single calibration. Comparison of data recorded at different times show that there are slight systematic variations in the measurements. We attribute these variations to minor modifications on the surface of the wire (e.g. due to dust or oxidation) and environmental influences such as small temperature changes.

We have also fitted our calibration results to (2) in terms of Nu_m using the calculational procedures described in §3.7. Figure 4 shows examples of Nu_m plotted as a function of $Re^{0.43}$ for all of the gases investigated. Note that helium has been included on the plot even though the value of n_0 for this gas is less than 0.30. Values of A_m and B_m for fits of (2) using $n = 0.43$ are included in table 4. These values refer to the particular data sets that are displayed in figure 4. The approximate Re -range of the measurements is also included in this table.

Equations (14)–(17) and (29) have been used to calculate Nu_∞ for each bridge-voltage reading recorded during a given pure-gas calibration. Examples of the results of these calculations for each gas are shown in figure 5 where Nu_∞ is shown plotted against $Re^{0.43}$. The data for each gas have been fitted to (2). Table 4 lists the resulting values of A_∞ and B_∞ .

The following observations concerning the effects of end conduction corrections on the calculated Nu -behaviour are obtained by comparing Nu_m and Nu_∞ :

- (i) Nu_∞ is always less than Nu_m . For this reason $A_\infty < A_m$ and $B_\infty < B_m$;
- (ii) the fractional decrease of B_∞ compared with B_m is much less than the corresponding decrease of A_∞ compared with A_m ;
- (iii) the value of n_0 is increased slightly by the correction, but, in general, $\Delta n_0 < 0.01$;
- (iv) the larger the k -value for the gas, the smaller is the relative reduction of Nu_∞ from the value of Nu_m ;
- (v) the relative decrease of Nu_m at higher velocities is slightly reduced from that found at lower velocities.

It should be noted that the calculational result that n_0 remains essentially constant while A_m and B_m both increase with end conduction losses is consistent with the experimental conclusions of Andrews *et al.* (1972) and others (Collis 1956; Champagne & Lundberg 1966). The relative changes in the magnitudes of A_m and B_m after these corrections are also consistent with these past experimental results.

Nu_∞ data shown in figure 5 have been corrected for thermal-slip effects assuming $\alpha = 1$, as described in §3.7. Values of n_0 along with A_c and B_c for $n = 0.45$, which result from fits of (2), are listed in table 5. In general, n_0 values for these fits are slightly lower than 0.45. However, we have chosen to treat our data in this form to be consistent with Collis & Williams (1959) as well as a large number of other workers. Differences in the quality of fits using actual values of n_0 or $n = 0.45$ will be very small for every gas except helium. Figure 6 shows plots of Nu_c versus $Re^{0.45}$ for the nine gases studied.

For some gases it has been necessary to consider accommodation effects. Calculations made for $\alpha \neq 1$ will be discussed in §5.

By comparing figure 3 with figures 4, 5 and 6 and the numerical results listed in tables 4 and 5, it is clear that the use of (2) in terms of either Nu_m , Nu_a , or Nu_c gives a rough correlation of the hot-wire data for different gases. It is also clear that this

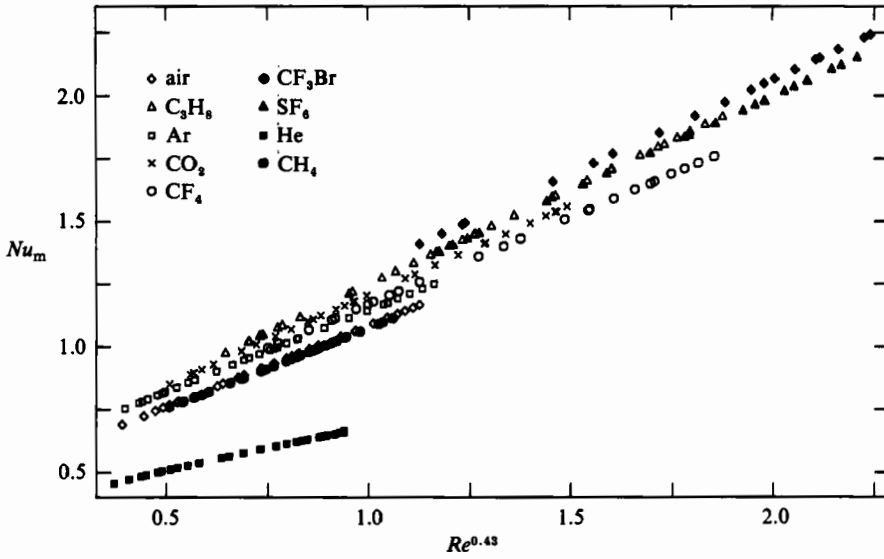


FIGURE 4. Representative values of measured Nusselt number (Nu_m) calculated using (28) are plotted as a function of $Re^{0.43}$. Results for nine different gases are included. With the exception of the helium results, the data used to calculate Nu_m are those shown in figure 3.

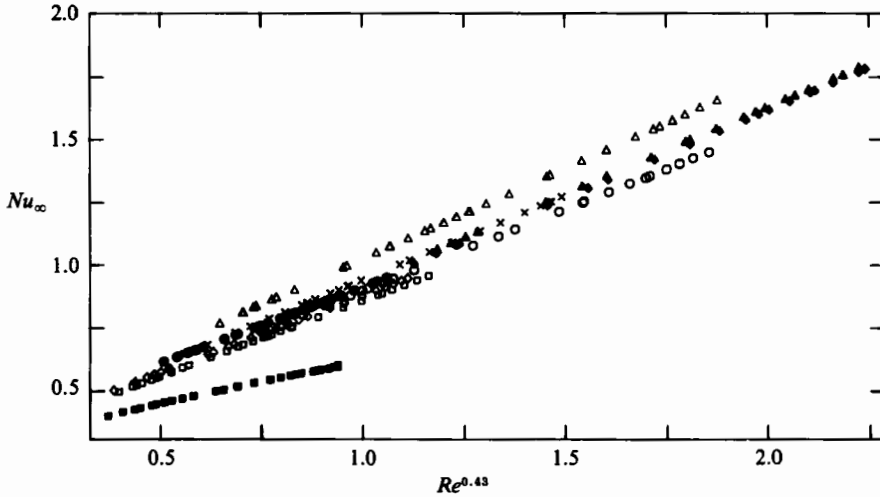


FIGURE 5. The results shown in figure 4 have been corrected for the finite aspect ratio of the hot wire. The resulting Nusselt numbers (Nu_∞) are plotted as function of $Re^{0.43}$. Symbols as in figure 4.

correlation is not complete and that other factors must be considered. This point will be discussed further in §5.

Note that the highest Re -value reached for the hot-wire measurements ($Re = 6.2$) was well below that required for the onset of vortex shedding ($Re \approx 44$).

4.2. Hot film

Calibrations for the hot film have been performed using procedures identical with those described for the hot wire. We have investigated the hot-film response for eight pure gases. Figure 7 shows examples of the hot-film data for different gases plotted

Gas	n_0	A_c	B_c	n_0	A_a	B_a	$A_a - (K_A)_x$	$B_a / (K_B)_x$
Air	0.44	0.272	0.650	0.44	0.272	0.650	0.272	0.650
CH ₄	0.46	0.318	0.635	0.47	0.312	0.679	0.282	0.640
Ar	0.44	0.261	0.660	0.44	0.261	0.660	0.274	0.644
C ₃ H ₈	0.45	0.322	0.725	0.45	0.322	0.725	0.274	0.651
CO ₂	0.44	0.282	0.698	0.44	0.282	0.698	0.272	0.643
He	0.35	0.289	0.410	0.45	0.281	0.630	0.284	0.646
SF ₆	0.43	0.264	0.695	0.43	0.264	0.695	0.256	0.640
CF ₃ Br	0.43	0.252	0.690	0.43	0.252	0.690	0.244	0.654
CF ₄	0.44	0.270	0.659	0.44	0.270	0.659	0.269	0.636

TABLE 5. Values of n_0 are listed which result when the hot-wire data for the different gases are fitted to (2). Values of A_c , B_c , A_a and B_a are reported using $n = 0.45$ in (2). The last two columns list the values of A and B which result when A_a and B_a are modified using the $(K_A)_x$ and $(K_B)_x$ values as defined by (32) and (33) respectively

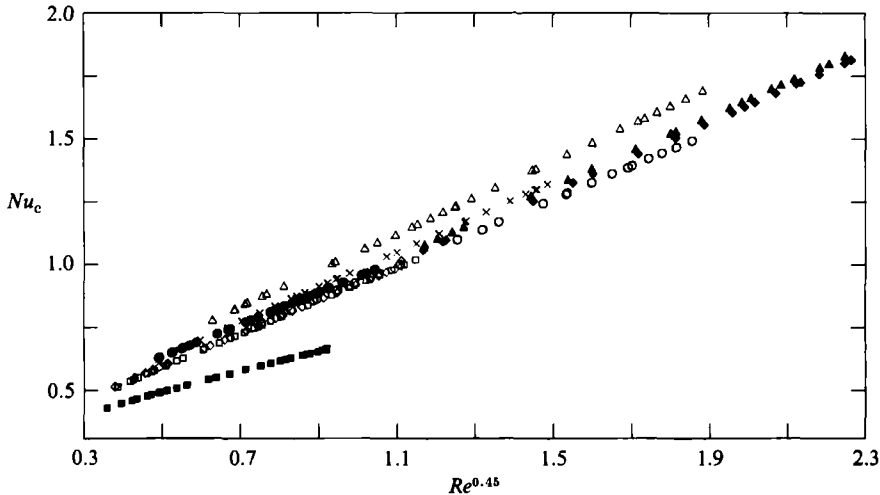


FIGURE 6. The results shown in figure 5 have been corrected for 'classical' rarefied gas effects. The resulting Nusselt number (Nu_c) corresponds to that expected for an infinite hot wire placed in a continuum gas. Accommodation effects are assumed to be negligible. Values of Nu_c are plotted as a function of $Re^{0.45}$. Symbols as in figure 4.

as E^2 versus $U^{0.45}$. Data for helium have been omitted from the figure for the same reasons as described above. Comparison of figures 3 and 7 shows that there are similarities in relative voltage magnitudes and slopes for the same gas on the hot wire and film. Further inspection reveals some striking differences. All of the hot-wire data was fitted quite accurately by using (26) with $n = 0.43$. A velocity exponent of 0.45 gives an accurate fit for some of the hot-film results (e.g. air and Ar), but there is a clear curvature in the plots of some of the other gases (e.g. C₃H₈ and SF₆) which indicates that different values of n would give an improved fit of the data to (26).

The data for all of the different gases investigated have been fitted to (26). Table 6 summarizes values of n_0 , A' , and B' for all of the hot-film measurements we have made. A' and B' are the parameters for least-squares fits taken with $n = 0.45$. Note that there is excellent agreement between the results of measurements for the same gas made over a period of several weeks. Comparison of n_0 values shows that

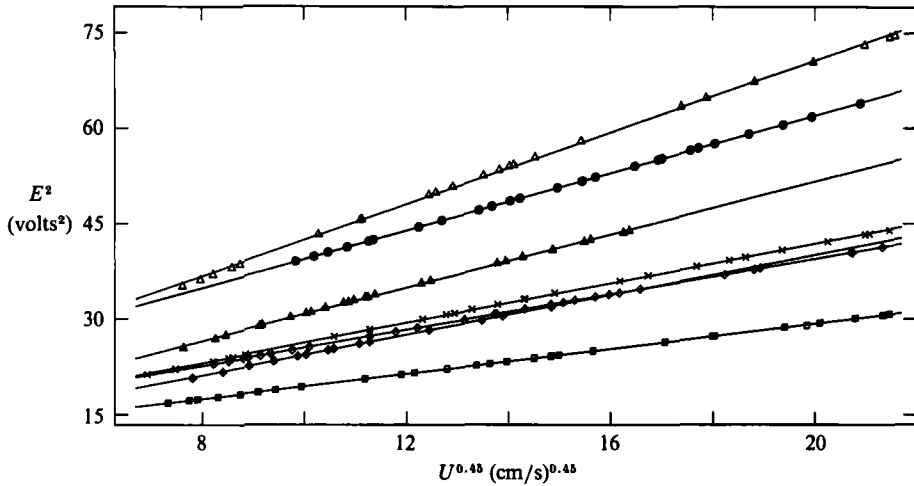


FIGURE 7. Representative data are shown for the response of a hot film to flows of seven different gases. The experimental results are plotted as E^2 (volts²) versus $U^{0.45}$ (cm/s)^{0.45}. Solid lines correspond to linear least-squares-curve fits of the data to (27) with $n = 0.45$. Note the curvature in the plots for SF₆, CF₃Br and C₃H₈. The curvature indicates that 0.45 is not the best value for n for fitting data of these gases to (2). Symbols as in figure 4.

Gas	n_0	A'	B'	Re-range	A_m	B_m	A_c	B_c
Air	0.44	12.04	1.387	1.3–13.5	1.089	0.854	0.434	0.807
	0.45	11.82	1.398	1.6–13.4				
	0.45	11.69	1.392	1.6–12.5				
CH ₄	0.42	17.36	2.284	2.6–12.9	0.963	0.892	0.529	0.847
	0.44	17.15	2.251	2.1–11.2				
Ar	0.45	9.91	0.981	1.4–14.8	1.362	0.860	0.416	0.814
C ₃ H ₈	0.37	14.99	2.833	7.0–42.8	1.247	0.962	0.592	0.911
	0.38	14.46	2.816	4.3–43.3				
	0.39	14.50	2.859	4.0–42.8				
CO ₂	0.41	11.00	1.547	1.5–25.0	1.272	0.926	0.457	0.872
	0.41	11.00	1.551	1.9–23.8				
He	0.39	40.34	2.550	1.0–2.5	0.694	0.730	0.506	0.738
SF ₆	0.37	10.68	2.083	14.4–40.2	1.447	0.899	0.448	0.843
	0.35	10.15	2.086	7.9–38.4				
CF ₃ Br	0.38	8.91	1.568	8.2–34.0	1.734	0.935	0.414	0.875

TABLE 6. This table lists results for the hot-film calibration in flows of different gases. Values of n_0 for fits of the data to (26) are listed along with the corresponding values of A' and B' found for $n = 0.45$. The Re-range covered by each set of measurements is also included. The last four columns contain selected A_m , B_m , A_c and B_c values which result from fits of the data in terms of Nu_m and Nu_c to (2) with $n = 0.45$

the curvature observed in the data for different gases in figure 7 is related to variations in this parameter. For the hot-wire results, only helium gave an n_0 value which differed significantly from 0.43. However, for the hot film, values of n_0 vary from 0.37 to 0.45. This variation in n_0 complicates the comparison of the hot-film response in different gases.

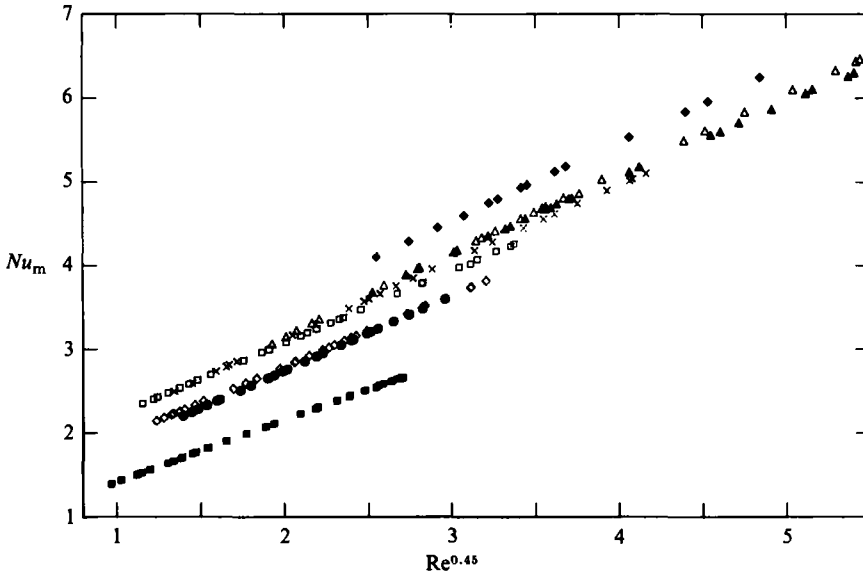


FIGURE 8. Representative values of measured Nusselt numbers Nu_m for the response of the hot film in different gases are plotted as a function of $Re^{0.45}$. The values of Nu_m correspond to the experimental data shown in figure 7 plus a data set recorded for flows of helium gas. Symbols as in figure 4.

Values of Nu_m as a function of Re have been calculated for the hot-film results using (28). Examples of the results for different gases are shown in figure 8, plotted as Nu_m versus $Re^{0.45}$.

Table 6 includes the values of A_m and B_m that give the best fit of (2) with $n = 0.45$ for selected data sets, together with the approximate Re -ranges over which the measurements for different gases have been made. A comparison of these ranges with the observed values of n_0 indicates that those gases for which the data extend to higher Re have smaller values of n_0 . As will be discussed in §6.3, we attribute this behaviour to transitions in the flow behaviour of the gas as it passes over the heated cylindrical film.

A dramatic demonstration of the importance of these transitions can be found by making the hot-film measurements for C_3H_8 , CF_3Br and SF_6 at slightly higher velocities than employed for the data reported above. Figure 9 shows results of measurements made over the Re -regime where the flow transition occurs. A clear change in heat-transfer behaviour can be seen. Least-squares fits generated from the lower-velocity data clearly give poor fits to the data recorded at flow velocities corresponding to Re which lie above the flow transition region. In an Re -range extending approximately from 36–55 a distinct hysteresis is observed which depends on whether the flow velocity is increased or decreased to the measurement value. Figure 10 shows plots of such measurements for the three gases. The source of this hysteresis will be discussed in §6.3; however, it is worthwhile to point out here that it occurs over an Re -regime where a flow transition ($Re \approx 44$) was observed by Collis & Williams (1959) and attributed by those authors to the onset of vortex shedding.

The hot-film results have also been corrected for end-conduction losses and rarefied-gas effects using the calculational procedures given in §3.7. It has been assumed that $\alpha = 1$. The corrections due to slip effects are very small. Figure 11

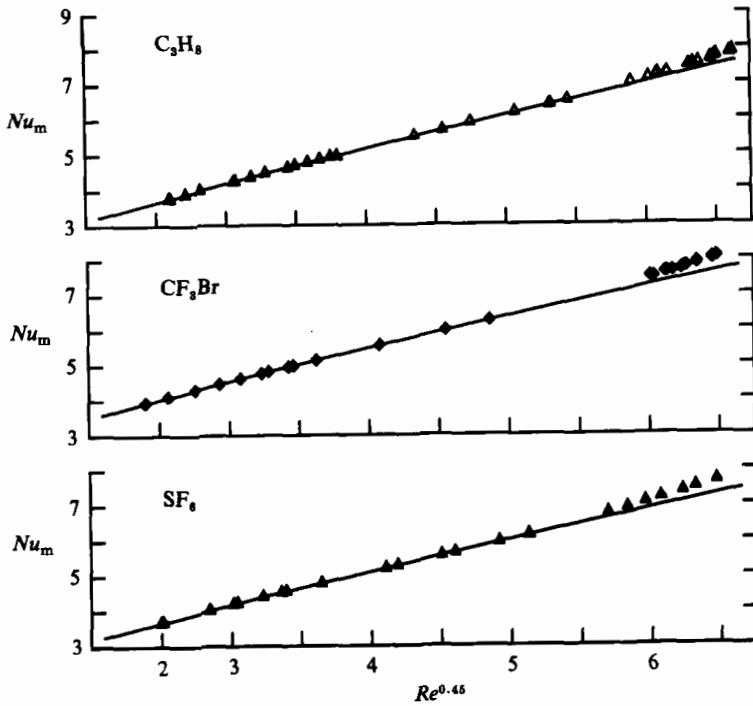


FIGURE 9. Hot-film results are plotted as Nu_m versus $Re^{0.45}$ for C_3H_8 , SF_6 and CF_3Br . These results span the Re -range ($Re \approx 44$) where the onset of vortex shedding from the heated cylinder is known to commence. This flow transition results in the markedly different heat-transfer behaviour observed in the high- and low- Re regions.

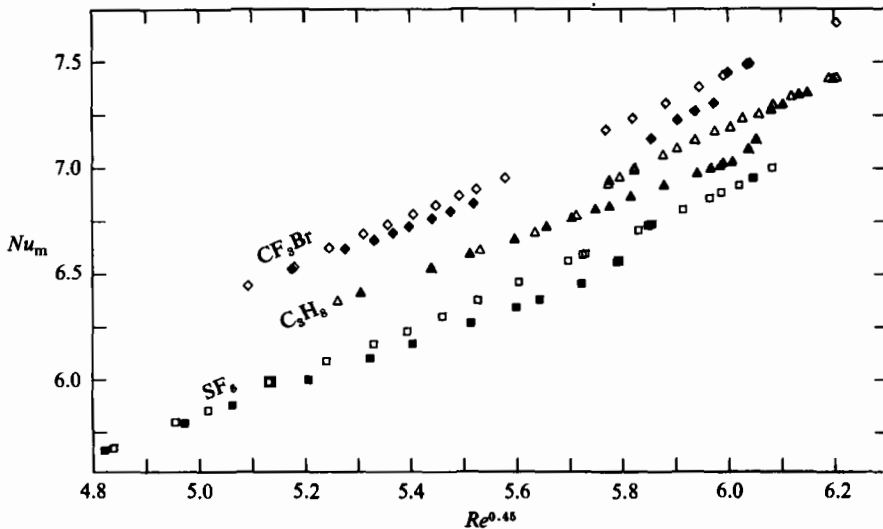


FIGURE 10. Values of Nu_m versus $Re^{0.45}$ are plotted for C_3H_8 (Δ), SF_6 (\square), and CF_3Br (\diamond) over the Re -range where the onset of vortex shedding from cylinders is known to occur. The measurements show a clear hysteresis in the data depending on whether the gas flow velocity is increased to the measurement value (denoted by filled symbols) or decreased to the measurement value (denoted by open symbols). For reasons of clarity, Nu_m values for SF_6 have been offset downward by 0.188 and those for CF_3Br have been offset upward by 0.063.

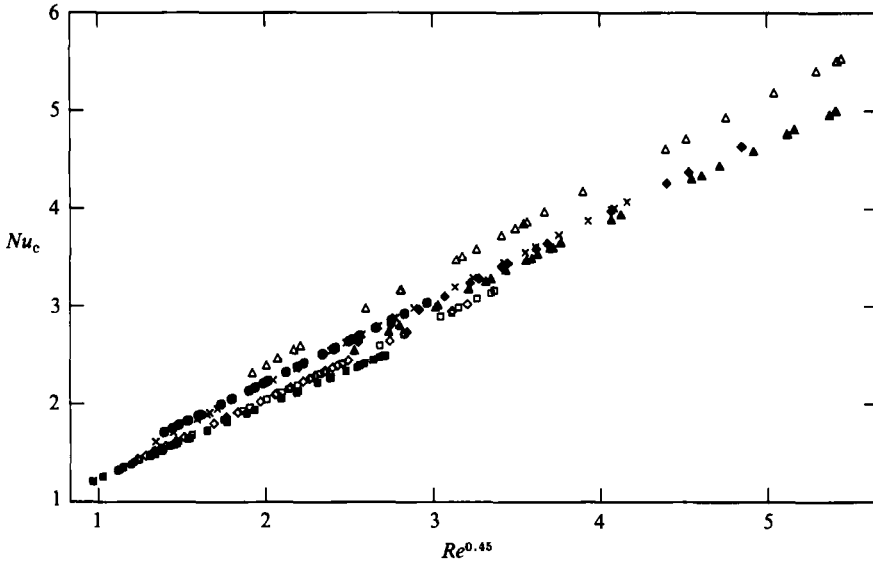


FIGURE 11. Values of Nu_c versus $Re^{0.45}$ are shown for the hot film. The results given in figure 8 have been corrected for finite aspect ratio and rarefied-gas effects to give the data shown here. Symbols as in figure 4.

shows examples of Nu_c plotted as a function of $Re^{0.45}$. Values of A_c and B_c from least-squares fits of the data to (2) are included in table 6.

As for the hot-wire results, it is found that (2) alone is not sufficient to correlate the hot-film results. The similarities between figures 4 and 8 and figures 6 and 11 are encouraging in that they indicate that the major source of variation in Nu -behaviour between different gases is due to gas properties and not to such variables as device response (two different devices were used), T_s (the wire and film were operated at different T_s), or Re -regimes (owing to the different diameters of the wire and film the Re ranges of the measurements vary widely).

5. Correlation of experimental results

The experimental results shown in figures 6 and 11 and summarized in tables 5 and 6 show that the hot-wire and -film responses in different gases are not correlated by simply plotting Nu_c versus Re^n . We have tried many schemes for collapsing our data for gases to single curves.

Two different philosophies have been employed in the literature to correlate heat-transfer data such as that generated in this study. The first assumes that an expression in the form of (2) can be used to fit all of the data if the proper corrections can be found for the coefficients A and B . The modification of A and B in (19) to allow for Pr -dependencies is an example of this approach. The second philosophy assumes that a simple correction should be applied to Nu in order to collapse the experimental data to a single curve. The temperature correction included in (3) and the ν -correction included in (21) are typical examples of this approach. Both philosophies have been tried in this work, as described below.

The first attempt to correlate the hot-wire experimental data assumed an equation of the form of (19), i.e. a Prandtl-number correction. This equation has been suggested as a means of correlating hot-wire results for different gases by several

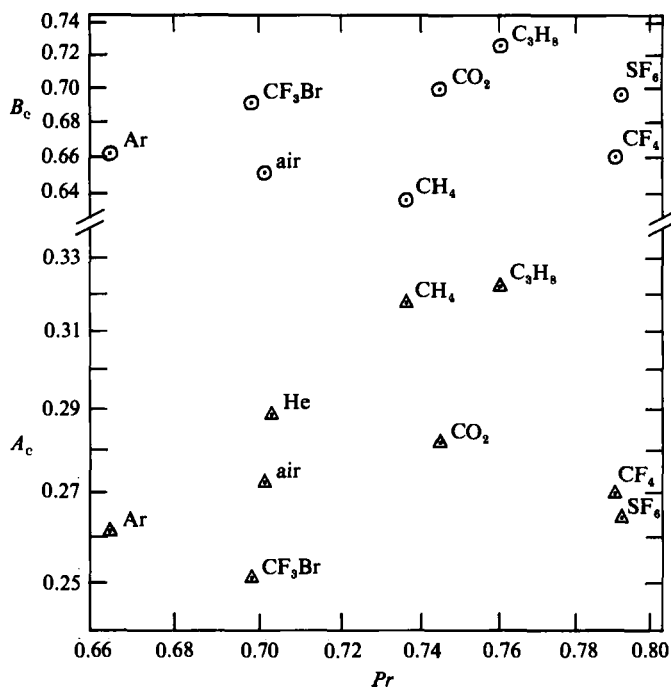


FIGURE 12. The parameters A_c (Δ) and B_c (\odot) which result from fits of the hot-wire data for nine different gases to (2) are plotted on a log-log plot as a function of Pr . Note that the value of B_c for helium is not included on the plot. This figure shows that there is no apparent dependence of these parameters on Pr .

authors (Wu & Libby 1971; McQuaid & Wright 1973; Simpson & Wyatt 1973). Figure 12 shows log-log plots of A_c and B_c as functions of Pr for the various gases. If (19) provided a means of correlating the measurements, these sets of data would form two straight lines. It is clear from figure 12 that this is not the case. Apparently, the use of (19) alone is not sufficient to correlate our experimental results. Note that we are omitting helium from consideration at this point owing to the large accommodation effects expected in heat transfer from the hot wire to this gas.

We next considered other means for reducing A_c and B_c values for the different gases to universal coefficients. The approach used is demonstrated in figures 13 and 14 which show log-log plots of A_c and B_c as functions of μ_m and ν_m/ν_∞ respectively. Note that the B_c value of helium of 0.410 has been omitted from these plots. If the coefficient data fall on straight lines, then the dependent variable provides a means to correlate the coefficients. Many different expressions for the dependent variable were tried. These included k_m , ν_m , and λ as well as those shown in figures 12–14. Most of these plots indicated that A_c and B_c had little or no dependence on these variables. However, as shown in figure 14, values of B_c were fitted quite well by assuming a power-law dependence of B_c on values of ν_m/ν_∞ for the different gases. Only the results for methane seem to be in disagreement with this conclusion and it will be shown that this is most likely due to a small accommodation effect. B_c was also found to be roughly correlated with k_m/k_∞ , but the correlation with ν_m/ν_∞ was slightly better. As shown in figure 13, values of A_c were found to be dependent on μ_m . Values of A_c for all nine gases seem to be reasonably well correlated using this parameter.

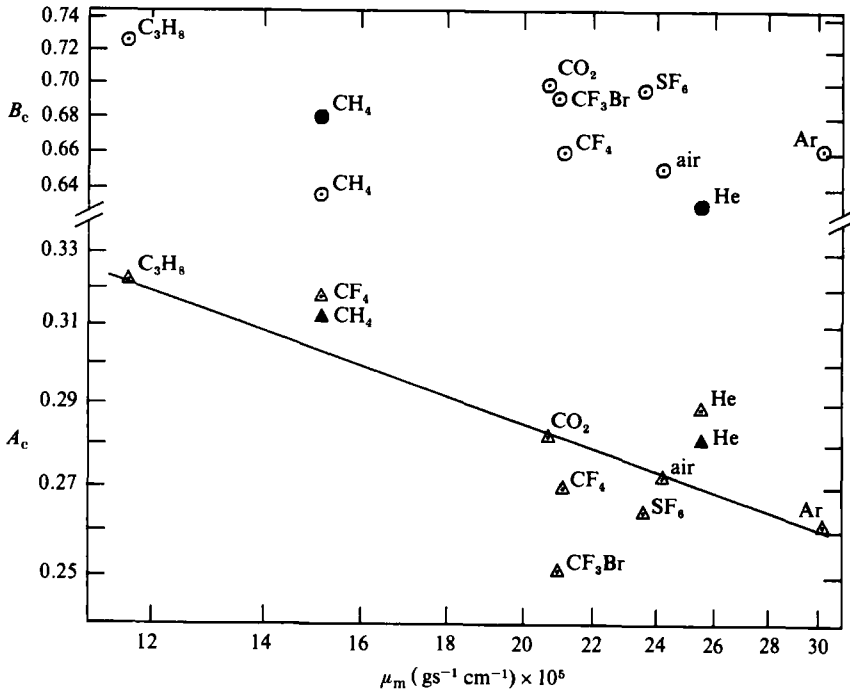


FIGURE 13. The parameters A_c (Δ) and B_c (\odot) found for the hot-wire measurements made in nine different gases are shown on a log-log plot as a function of $(\mu_m)_x$. Note that B_c for helium has been omitted. This plot indicates a dependence of A_c on $(\mu_m)_x$. The straight line is the result of a linear least-squares curve fit of $\log(A_c)$ versus $\log(\mu_m)_x$. Results of this fit are used to give (31). The filled symbols correspond to values of A_a and B_a calculated for helium and methane.

Since the molecular properties of air, Ar, CO_2 and C_3H_8 are believed to be more accurately known than those for the other gases, the results for measurements in these four gases have been used to calculate expressions for A_c and B_c in terms of the suitable dependent variable. The results are

$$(B_c)_x = 0.2606 \left(\frac{\nu_m}{\nu_\infty} \right)_x^{1.355}, \quad (30)$$

and

$$(A_c)_x = \frac{0.0431}{(\mu_m)_x^{0.222}}, \quad (31)$$

for gas x . Equations (30) and (31) are taken from linear least-squares fits of the logs of the independent and dependent variables. The lines corresponding to these fits are included in figures 13 and 14.

Equations (31) and (32) have been used to develop a correlation of the experimental results for the different gases. Instead of using these equations directly, we have normalized the results to those of air by plotting $Nu_c - (K_A)_x$ versus $Re^{0.45}(K_B)_x$ for $(K_A)_x$ defined as

$$(K_A)_x = (A_c)_{\text{air}} - [(\mu_m)_{\text{air}}/(\mu_m)_x]^{0.222}(A_c)_{\text{air}} \quad (32)$$

and $(K_B)_x$ as

$$(K_B)_x = \left[\frac{(\nu_m/\nu_\infty)_x}{(\nu_m/\nu_\infty)_{\text{air}}} \right]^{1.355} \quad (33)$$

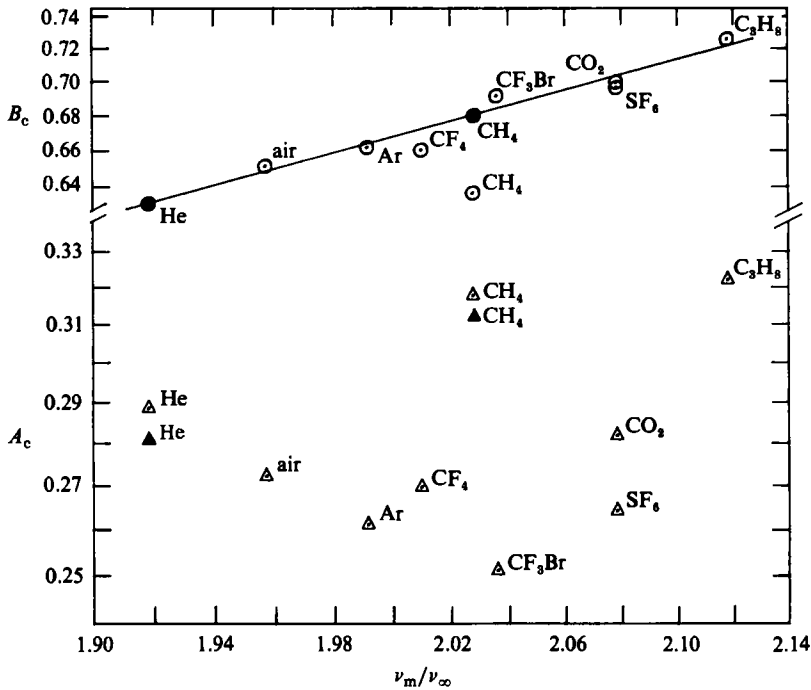


FIGURE 14. The parameters A_c (Δ) and B_c (\odot) found for the hot-wire measurements made in nine different gases are shown on a log-log plot as a function of $(\nu_m/\nu_\infty)_x$. Note that B_c for helium has been omitted. This plot indicates a dependence of B_c on $(\nu_m/\nu_\infty)_x$. The straight line is the result of a linear least-squares curve fit of $\log(B_c)$ versus $\log(\nu_m/\nu_\infty)_x$. Results of this fit are used to give (30). The filled symbols correspond to values of A_a and B_a calculated for helium and methane.

Figure 15 shows the data plotted in this manner. It is clear that this calculational procedure provides a good correlation for all of the data except that for helium and methane.

In the past, deviations in the heat-transfer behaviour of heated cylinders in flows of helium have been attributed to the lack of 'accommodation' of this light gas on the surface. This effect can be treated using (8)–(11) and allowing α to be a parameter. Equation (10) is then used to generate new values of Nu_a from Nu_c where ϕ is now a function of α . Values of Nu_a are then fitted to (2). In order to determine the best value of ϕ for use in (10), it has been assumed that B_a equals the B_c predicted for helium using (31). For helium, this was found to be true for $\alpha = 0.48$.

Significantly, n_0 is found to be 0.45 when Nu_a values for helium are fitted to (2) by the linear least-squares fitting procedure. This result is in excellent agreement with the values of n_0 found for measurements in the other gases. Contrast this value of n_0 with that of $n_0 = 0.35$ which results from a plot of the helium results as Nu_c versus Re^n .

Values of A_a and B_a for helium with $\alpha = 0.48$ are plotted as solid symbols in figures 13 and 14. As required by the calculational procedure, B_a lies on the line corresponding to (30) which is included in figure 14. There is also good agreement between the value of A_a and that predicted by (31), as can be seen in figure 13.

As noted earlier, the value of B_c (see figure 14) for methane is slightly lower than predicted by (30). We have assumed that this observation is due to a slight accommodation effect for methane on the probe surface and have treated the data

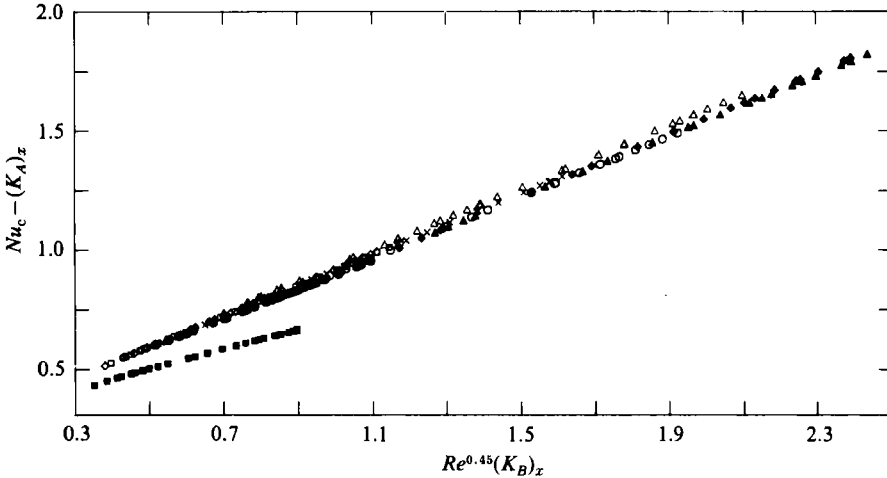


FIGURE 15. The correlation of the experimental results which is obtained when the values of Nu_c for the hot-wire in different gases are corrected for variations in (ν_m/ν_∞) and μ_m is shown. Values of $Nu_c - (K_A)_x$ are plotted as a function of $Re^{0.45}(K_B)_x$. $(K_A)_x$ and $(K_B)_x$ are given by (32) and (33) respectively. With the exception of the results for helium, an excellent correlation of the data is obtained. Symbols as in figure 4.

in the same manner as for helium. In this case, B_a is found to equal the predicted B_c for $\alpha = 0.61$. The value of $n_0 = 0.47$ found for the plot of Nu_a versus Re^n is a little higher than expected, but is not unreasonably larger than the values of 0.44 and 0.45 found for the other gases. Values of A_a and B_a for methane are included in figures 13 and 14 as solid symbols. There is good agreement between these values and those predicted using (30) and (31).

Figure 16 shows the effects of correcting Nu_c for accommodation effects on the collapse of the experimental data to a single curve. Values of $Nu_a - (K_A)_x$ with $\alpha = 0.48$ for helium, $\alpha = 0.61$ for methane, and $\alpha = 1$ for the remaining seven gases are plotted as a function of $Re^{0.45}(K_B)_x$. Collapse of the experimental data to a single curve is excellent. Corrected values of A_a and B_a for the nine gases are listed in the last two columns of table 5. The calculational procedure has clearly correlated the heat-loss data for the nine different gases to a high degree.

Correlations similar to (21) have also been tried. Ratios of (ν_m/ν_∞) and (k_m/k_∞) have been used as fitting parameters. The exponents necessary to generate the best fits have been treated as variables. They have been generated by making log-log plots of calculated values of Nu_c at $Re = 1$ against $(\nu_m/\nu_\infty)_x$ where x is air, C_3H_8 , CF_4 , CF_3Br , CO_2 , Ar or SF_6 . A linear least-squares fitting procedure gives the exponent from the slope of the line. For ν_m/ν_∞ the exponent was found to be 1.48. This value is considerably larger than the value of 0.15 given by Fingerson & Ahmed (1976) for a cooled film. The exponent for use with (k_m/k_∞) is calculated to be 0.274.

Figure 17 shows $Nu_c(K_v)_x$, where

$$(K_v)_x = \left[\frac{(\nu_m/\nu_\infty)_{\text{air}}}{(\nu_m/\nu_\infty)_x} \right]^{1.48}, \quad (34)$$

plotted against $(Re)^{0.45}$. As above, the data for the different gases have been normalized to the results for air. This procedure does give a fairly good correlation for all of the gases except helium, for which the fit could be improved using an $\alpha \neq 1$

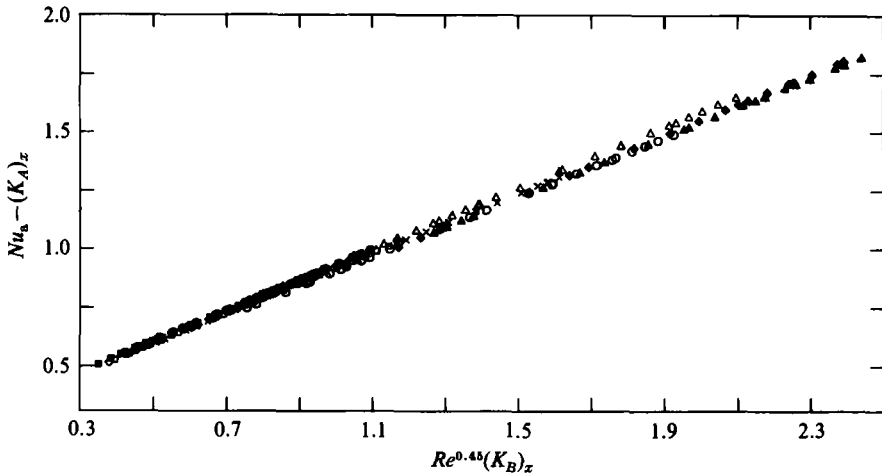


FIGURE 16. This correlation of experimental results is the same as that given in figure 15 with the exception that the helium and methane results have been corrected for accommodation effects. The correlation of results for measurements made in all nine gases is now found to be excellent. Symbols as in figure 4.

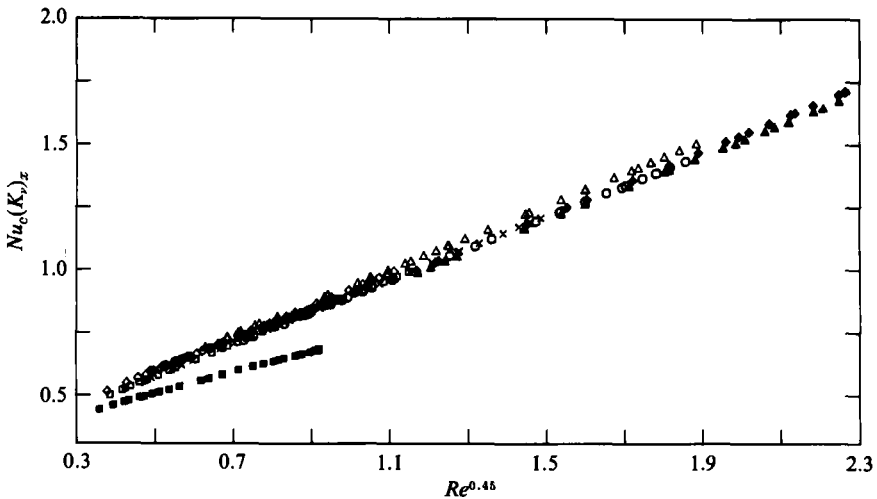


FIGURE 17. Values of Nu_c measured for the hot-wire are multiplied by $(K_v)_x$ (as given by (34)) and plotted as a function of $Re^{0.45}$. The results for helium have not been corrected for accommodation effects. Symbols as in figure 4.

as was done above. By comparing figures 16 and 17, it is clear that the calculational procedure which modifies the parameters A_c and B_c as given by (30) and (31) provides a much better correlation of the experimental results.

Figure 18 shows $Nu_c(K_k)_x$ plotted as a function of $Re^{0.45}$. $(K_k)_x$ is defined analogously to $(K_v)_x$, namely

$$(K_k)_x = \left[\frac{(k_m/k_\infty)_{air}}{(k_m/k_\infty)_x} \right]^{0.274} \tag{35}$$

Plotting the experimental data in this way also gives an improved correlation of the data for the different gases compared with using Nu_c alone. However, the degree of

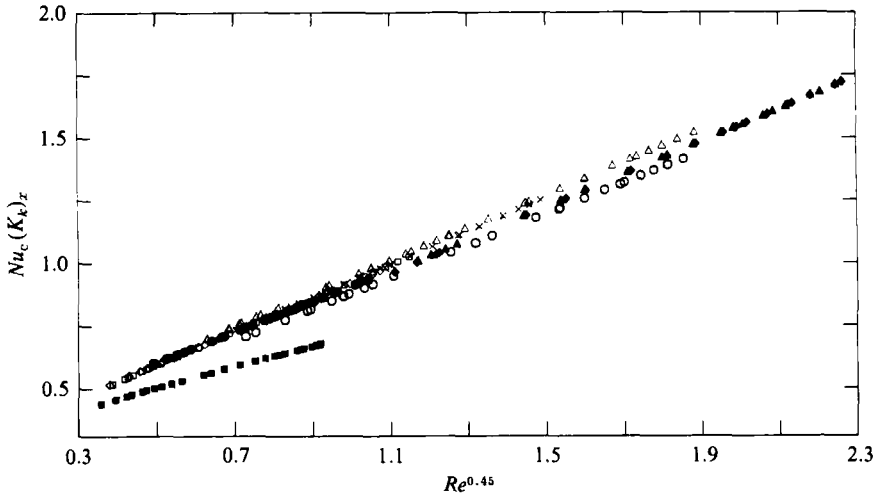


FIGURE 18. Values of Nu_c measured for the hot wire are multiplied by $(K_k)_x$ (as given by (35)) and plotted as a function of $Re^{0.45}$. The results for helium have not been corrected for accommodation effects. Symbols as in figure 4.

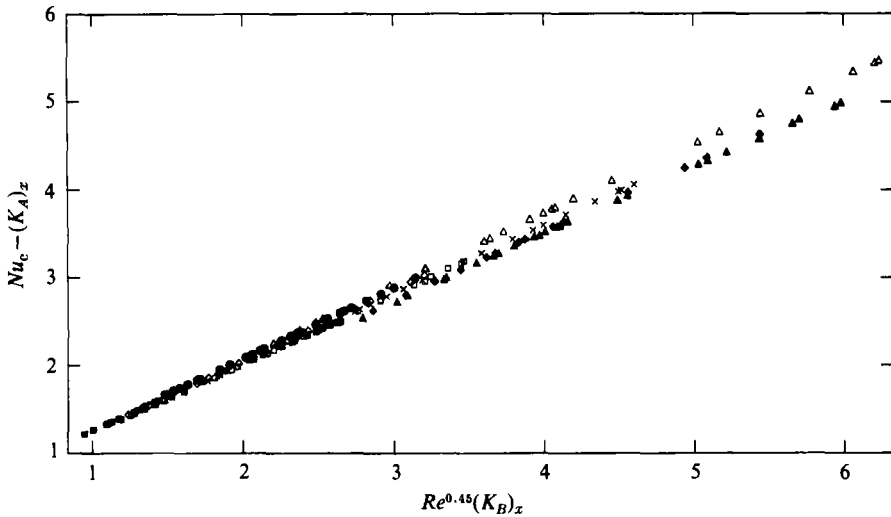


FIGURE 19. Values for $Nu_c - (K_A)_x$ versus $Re^{0.45} (K_B)_x$ are plotted for the eight different gases investigated with the hot film. Expressions for $(K_B)_x$ and $(K_A)_x$ are the same as those used for the hot wire (i.e. (32) and (33)). Symbols as in figure 4.

correlation can be seen to be slightly less than shown in figure 17, while figure 16 still seems to provide the best collapse of the data.

Attempts to correlate the results for the hot-film response in different gases is complicated by the variation of n_0 when the experimental data are fitted to (2). For this reason it was decided to treat the hot-film data in the same way as described above for the hot-wire data. The hope was that such a treatment would collapse the hot-film data for different gases to a single curve. Figure 19 shows $Nu_c - (K_A)_x$ versus $Re^{0.45}(K_B)_x$ for the eight gases for which hot-film response measurements have been made. Here $(K_A)_x$ and $(K_B)_x$ are given by (32) and (33) with T_m taken as 472 K. The data plotted in this figure show that the hot-film response for different gases is

correlated well by treating the data in this manner. However, the collapse of the data to a single curve is not as complete as found for the hot-wire data. Note that it is not necessary, as is the case for the hot-wire, to modify calculated values of Nu_c of He and CH_4 for effects due to a lack of gas accommodation on the surface of the hot-film.

The fact that the same correlation procedure works for the hot-wire and hot-film results implies that the procedure is independent of probe-surface-temperature effects, aspect ratio and probe diameter since these parameters are very different for the two probes investigated here. The implications of this finding will be discussed further in the next section.

6. Discussion

6.1. Correlation of hot-wire and hot-film responses in different gases

6.1.1. Precision of correlations

Figures 16, 17 and 18 show that it is possible to collapse the experimental data for the response of a hot-wire in different gases to a single line with a high degree of accuracy. For each of these plots, the data for different gases are normalized by suitable correction procedures to the results for air plotted as Nu_c (or Nu_a) against $Re^{0.45}$. Note that K_A , K_B , K_ν , and K_k are all equal to one for air. The line which results from fitting the air results to (2) with $n = 0.45$ is

$$Nu_c = 0.272 + 0.650 Re^{0.45}. \quad (36)$$

Analysis of the data shown in figure 16 indicates that the largest deviations from this line for the data of other gases is +2.0% and -2.7%. As can be seen in figure 16, it is the data for CF_4 , CF_3Br and SF_6 which are in poorest agreement with (36). On the other hand, the data for C_3H_8 , CH_4 , Ar and CO_2 all fall within 1% of the values predicted by (36).

A similar analysis shows that the method of correlation used for the data displayed in figure 17 gives results which all lie within +3.6% and -4.5% of the line given by (36). Clearly the deviations of the data from (36) are greater for data calculated by multiplying $(Nu_c)_x$ by $(K_\nu)_x$ than using the two-parameter correction employed for the results shown in figure 16. Note that, as drawn in figure 17, the Nu_c values for helium and methane have not been corrected for accommodation effects. The error limits reported here refer to data corrected for these effects. The correction employed for the data shown in figure 17 corresponds to multiplying $(A_c)_x$ and $(B_c)_x$ by $(K_\nu)_x$. Comparison of (33) and (34) shows that there is very little difference between values of $(K_A)_x$ and $(K_\nu)_x$. For this reason, the increased scatter of the one-parameter correction is due primarily to improper correction of the intercepts $(A_c)_x$.

In figure 18 the maximum deviations found from the line given by (36) are +2.9% and -7.3%. As already noted in §5, the calculational procedure used for the data displayed in this figure yields the poorest correlation of the three used.

In discussing the precision of the various correlations for the hot-wire results, it must be kept in mind that there are uncertainties in the values of molecular properties used in the various calculations. Unfortunately, the largest uncertainties are found in the thermal conductivities for the different gases. These values are very important since they play a dominant role in end conduction-loss calculations as well as appearing to the first power in the Nu . The parameters A and B given in (2) are directly proportional to this variable.

For two of the gases, namely CF_3Br and C_3H_8 , widely different values of k are available in the literature. We have chosen to use the literature values of k which yield values of Nu_c consistent with those found for the other gases investigated. In the case of propane, this is a value taken from work (Ehya *et al.* 1972) published after the TPRC tables (Touloukian *et al.* 1970) had appeared. The later value of k is 5.6% times larger than the TPRC value. Interestingly, an uncertainty estimate of 5% is given in the TPRC for $k_{\text{C}_3\text{H}_8}$ at the temperature of interest to this study. In the case of CF_3Br we have used a value of k taken from an earlier (ASHRAE 1976) handbook and have ignored a later measurement (Geller *et al.* 1975) of $k_{\text{CF}_3\text{Br}}$ which gives a value which is 4.7% greater.

Given such uncertainties in k -values, we conclude that our calculational procedures yield correlations of hot-wire response data which are as precise as possible within the limits of our knowledge of gas properties. In fact, such uncertainties may explain the small variations of the data plotted in figure 16 from (36). It is the data for CF_3Br , SF_6 and CF_4 which deviate to the largest degree. These gases are also those for which the largest uncertainties in k -values are expected.

Comparison of the hot-film results plotted in figure 19 with those for the hot wire shown in figure 16 indicates clearly that the hot-film results are not correlated quite as well. There are several possible explanations for this observation. Perhaps the most likely one lies in the simplifying assumptions used to treat end heat-conduction losses for the hot film. This device has a very complicated physical structure and the assumption of a one-dimensional heat transfer is certainly naive. Furthermore, as pointed out in §3.7, k_s must be considered as an approximation of the true one-dimensional heat transfer conductivity. Since l/D is much smaller for the hot film than for the hot wire, end-loss corrections are expected to be more important in the case of the hot film.

We have assumed that the hot-film results can be corrected for gas-property temperature dependencies using the same procedures as for the hot wire (see (32) and (33)). Even though this assumption should be approximately correct, it is possible that the same correction may not apply if the hot-film surface has a different surface-temperature distribution or heat-loss behaviour. Uncertainties in the values of molecular properties may also be playing a role.

We conclude that correlating our hot-film results in the same manner as for the hot wire yields a collapse of the data that is precise to within the limits defined by our lack of detailed knowledge of heat conduction within the probe and uncertainties in the values of molecular properties for the different gases investigated.

6.1.2. *Physical reasonability of the correlation*

In developing the correlations of heat-transfer behaviour given in §5 we have made a variety of assumptions and approximations. It seems pertinent to try and understand the physical basis of these assumptions and also to relate them to past engineering practice.

The first choice we have made is to treat our experimental results in terms of non-dimensional variables. Reduction of experimental data in this way is standard engineering practice and has its basis in dimensional analysis (Kreith 1973). Problems of heat transfer to and from flows of gases are usually treated by experimentally determining variations of Nusselt number as a function of flow Reynolds number and other possible non-dimensionalized numbers such as the Prandtl number. One of the major limitations of this approach is that there are no definitive recipes for deciding how such non-dimensional variables should be formed.

For instance, in this study there were standard and physically based expressions for generating the appropriate Nu and Re (i.e. in terms of probe diameter), but there was no standard method for deciding at which temperature to evaluate the molecular properties used to form the non-dimensional variables. A wide variety of different temperatures have been used in the literature. We finally settled on an evaluation of properties at T_m not only because this temperature is widely used in the literature, but also because it seems reasonable from a physical point of view. The choice of a median temperature for molecular properties should minimize the effects of property variation with temperature.

We first attempted to correlate our results in terms of Nu_m . These attempts failed owing to the neglect of heat losses at the end of the probe. These conduction heat losses not only vary with the flow parameters, but are also dependent on the thermal conductivity of the gas. End-heat-loss corrections are discussed further in §6.2 where we conclude that the mathematical procedure used is physically realistic for the hot wire and is only approximately correct for the hot film.

Comparison of figures 4 and 5 shows that not only are values of Nu_∞ for the different gases greatly reduced from Nu_m by this correction, but the relative spacings and orderings of the data for different gases are changed. This is particularly true in the case of helium which has an unusually large value of k_m . For this reason, values of Nu_m for this gas are reduced proportionally less by this correction than found for the other gases. Similarly, CF_3Br has an extremely small k_m value and the reductions of its Nu_m values following end-loss corrections are proportionally greater than found for the other gases.

Correction of Nu_m for end conduction losses leads to a Nusselt number (Nu_∞) which corresponds to the Nu for convective heat transfer from an infinite cylinder. It is not surprising that the use of Nu_∞ yields a better correlation of experimental results than Nu_m .

Values of Nu_∞ are next corrected for what might be called 'classical' thermal slip. By this we mean non-continuum effects on heat transfer, but assuming perfect accommodation of the gas on the surface of the probe. Equations (8)–(11) with α set to 1 are used for this purpose. Figure 6 shows plots of the data for the nine gases plotted as Nu_c versus $Re^{0.45}$. Comparing values of Nu_c shown in figure 6 with the corresponding values of Nu_∞ shown in figure 5 indicates that the correction for non-continuum effects has very little effect on the relative ordering or spacing of the results for different gases. Note that there is a significant increase in the value of n_0 following the correction for thermal slip. As might be expected, the largest effect is for helium ($Kn = 0.075$) which has the largest Kn of any of the gases investigated here. Even for this gas, correction of our measurements for 'classical' thermal slip only increases values of Nu_c by approximately 10% compared with Nu_∞ .

As figures 6 and 11 show, plots of Nu_c as functions of $Re^{0.45}$ for the different gases do not lead to a complete correlation of the results. Clearly, some additional correction must be made. As discussed in §5, multiplying Nu_c by either $(k_m/k_\infty)_x$ or $(\nu_m/\nu_\infty)_x$ raised to suitable powers increases the degree of correlation of the results (see figures 17 and 18). Is this physically reasonable? The answer is yes. By multiplying Nu_c by either of these ratios, we are partially correcting values of Nu_c for variations in the temperature dependencies of the molecular properties of the gases. Heat transfer from a heated cylinder actually occurs along a temperature gradient; therefore it is not surprising that the temperature dependence of the molecular properties is important in the heat transfer. Since variations in k and ν for different gases are roughly related, it is expected that correlations in terms of

either k_m/k_∞ or ν_m/ν_∞ will give nearly the same collapse of the data. This is found to be the case.

The necessity of providing corrections for physical-property temperature dependencies is well known. The method used here is consistent with approaches which have been employed in past engineering studies (Kays & Crawford 1980).

Modification of $(Nu_c)_x$ by multiplying by either $(K_k)_x$ or $(K_\nu)_x$ gives a much better collapse of the data than is found for data simply plotted as $(Nu_c)_x$. However, by using equations of the same form as (2) and looking for ways to correlate the values of A_c and B_c individually, it has been possible to obtain an improved correlation of the results. The parameter B_c was found to be dependent on $(\nu_m/\nu_\infty)_x$ raised to a power (see figure 14 and (30)). This dependence is similar to that described above for the corrections of Nu_c for the temperature dependence of gas properties. Since the parameter B in (2) is generally thought to be determined by forced convection, it is reasonable to expect this parameter to show a similar dependence on molecular-property temperature variation.

On the other hand, the parameter $(A_c)_x$ has been shown to be primarily dependent on values of $(\mu_m)_x$ raised to a power (see figure 13 and (31)). While not exactly correct, this parameter is expected to be determined primarily by buoyant and conductive heat-transfer processes. It has not been possible to relate such heat-loss mechanisms to a dependence on μ_m . At present the physical basis of the observed dependence of $(A_c)_x$ on $(\mu_m)_x$ must be considered as unknown and perhaps coincidental.

It has been necessary to make one further correction of the data in order to obtain a satisfactory correlation of the experimental results for all of the gases. This final correction allows for accommodation effects (Kennard 1938). This effect will be discussed in greater detail in §6.4. Equations (8), (10) and (11), which have been used here to correct Nu_c for the effects of accommodation, provide a good approximation of the dependence of heat transfer on Kn and α that are found in more detailed theories of this effect.

In general, it is found that lighter gases show more pronounced accommodation effects than heavier gases (Devienne 1967). This is due to their smaller sizes and higher average molecular speeds. In this study we have only found it necessary to modify the hot-wire values of Nu_c measured in helium and methane in order to account for changes in transfer behaviour due to small values of α . This finding does not imply that small accommodation effects are not present for the other gases investigated, but only that such corrections are similar for measurements made in the other gases (see the discussion in §6.4).

In the case of helium it is highly significant that the correction for accommodation effects also changes the observed value of n_0 from 0.35 for the data in terms of Nu_c to the value of 0.45 found for the data treated as Nu_a . This is a clear indication that the correction of Nu_c for accommodation is behaving properly since the new value of n_0 is in complete agreement with those found for the other gases when plotted in terms of Nu_c . Further verification of this conclusion is provided by the results of the hot-film measurements in helium. For this case, no accommodation effects are required in order to correlate the helium data with that found for the other gases (see figure 19). This observation is in accord with the expectation that helium will be accommodated much better on a heated alumina surface (film) than on a heated platinum surface (wire) (Devienne 1967). The reduction in accommodation effects in the heat transfer of helium for a hot film compared with a hot wire has been discussed by Simpson & Wyatt (1973).

When developing the types of corrections for heat-transfer data we have used here

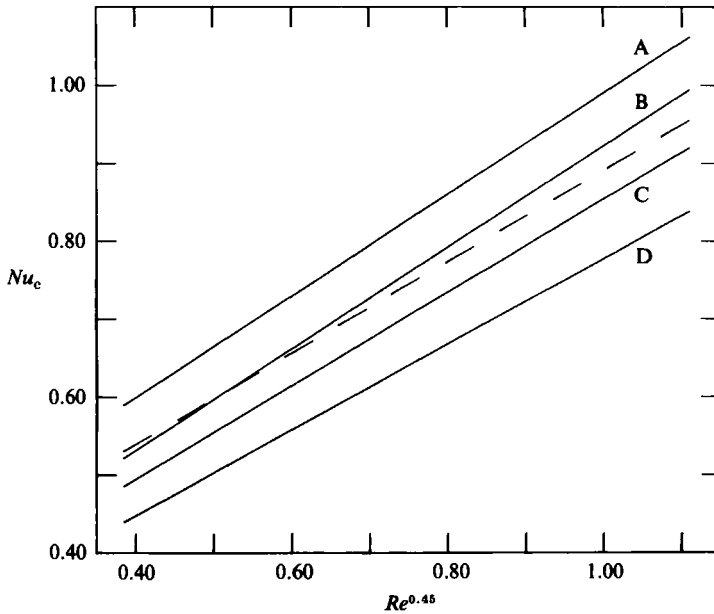


FIGURE 20. Correlations reported by Andrews *et al.* (1972), Collis & Williams (1959) and Fingerson & Ahmed (1976) are employed to give the solid lines (A, C and D respectively) shown as Nu_c versus $Re^{0.45}$ for the hot wire placed in a flow of air. T_m is assumed to be 431 K and T_∞ to be room temperature. The correlation that has been obtained in this work (36) is also included in the plot (line B). The dashed line is the result which is found if the correlation of Andrews *et al.* (1972) is modified to reflect more-recent literature measurements of the thermal conductivity of air than that used by Andrews *et al.* (see the discussion in the text).

it must be remembered that very simple equations are being used to approximate very complicated processes involving complex boundary layers and heat transfer at surfaces. Despite this, after correction of the experimental data for physically realistic heat-loss processes and known molecular effects a very good correlation of the heat-transfer data from heated filaments is found for a wide range of gases. This finding is the best justification for the various approximations and simplifications that have been employed.

6.1.3. Comparison of results for air with literature correlations

Most of the past work on correlations of heat transfer from anemometric probes has been reported for measurements made in air. Figure 20 shows the line corresponding to our results for air plotted as Nu_c versus $Re^{0.45}$. Note that the actual value of n_0 in this work was determined to be 0.44. This small change in n_0 has very little effect on the fit of the data to the equation. On the same plot the corresponding curves are drawn for results taken from Collis & Williams (1959) (3), Andrews *et al.* (1972) (12), and Fingerson & Ahmed (1976) (21). For (3) values of T_m and T_∞ correspond to those used in this study. Similarly, values of ν_m and ν_∞ used in (21) are taken for the conditions of the hot wire in our study. The two literature hot-wire studies employed probes having large aspect ratios while the cooled film used by Fingerson & Ahmed had a small aspect ratio. Apparently, the latter results have not been corrected for end heat-conduction effects.

Figure 20 shows that the present experimental results for Nu_c as a function of $Re^{0.45}$ lie very close to the curve given by Collis & Williams (1959). There is a

difference of approximately 9% in slope and 6% in intercept. Given the various corrections applied to our finite-aspect-ratio hot-wire measurements, this agreement must be considered excellent. As discussed in §2.1, variations in heat-transfer laws might also be expected due to the use of a feedback amplifier in our electronic circuitry (Bruun 1975; Morrison 1976).

The correlation reported by Andrews *et al.* (1972) is in poor agreement with our results (approximately the same slope and 25% higher in intercept) and those given by Collis & Williams (1959). A possible explanation for this discrepancy can be found by comparing the thermal-conductivity values for air used in the two earlier papers. Andrews *et al.* report that their k - and μ -values were taken from the International Critical Tables (1929) while Collis & Williams have used a formula for k given by Kannuluik & Carman (1951) and have taken their ν -values from the tables of Goldstein (1938). The values of kinematic viscosity employed in these two earlier studies are nearly identical with those used in this study. However, the values of k for air at T_m obtained from the literature sources for the three studies are 7.51×10^{-5} , 8.34×10^{-5} and 8.38×10^{-5} cal/cm K s for Andrews *et al.*, Collis & Williams, and this work respectively, i.e. the value of k used by Andrews *et al.* is 10% smaller than that used in this study or the work of Collis & Williams. The effect of using the more recent value of k on the results of Andrews *et al.* is shown in figure 20, where their results have been redrawn (dotted line) using our value of $(k_m)_{\text{air}}$. It can be seen that the results for all three studies of heat transfer from hot wires in flowing air now lie relatively close together. On this basis, we conclude that our measurements of Nu_c in air are consistent with past literature measurements.

The correlation of cooled-film results reported by Fingerson & Ahmed (1976) lies considerably below those measured for the hot wires. This apparent difference between the most suitable correlations for the hot-wire and cooled-film results has already been noted by Fingerson & Ahmed and attributed to a dynamical dissimilarity between heating and cooling of cylinders.

Figure 21 shows the line corresponding to our hot-film results in air plotted as Nu_c versus $Re^{0.45}$. Lines corresponding to the results of Collis & Williams (1959) and Andrews *et al.* (1972) (both as given by the authors and corrected for k -values as described above) are shown on the same plot. It is clear that there is relatively poor agreement between the hot-film results and the two earlier correlations obtained for hot wires. Both the slope and intercept for our data are higher. Using a larger value of k_s for the thermal conductivity of the film would result in decreased values of Nu_c when Nu_m is corrected for end conduction losses and non-continuum effects. Such a modification would give better agreement with the hot-wire correlations. However, as already noted in §4.1, the primary effects of the end conduction corrections are to generate a relatively large decrease in A_c and a much smaller decrease in B_c . Test calculations indicate that while reducing k_s does indeed increase the agreement of our hot-film results with those of Collis & Williams and Andrews *et al.*, this correction alone is not sufficient to give agreement for both A_c and B_c . We attribute this observation to the simplifications that have been made by assuming a one-dimensional heat conduction along the film. Improved agreement between heat-transfer laws for hot films and hot wires will require a better theoretical treatment of heat-conduction losses for the hot film.

6.1.4. Comparison of results for different gases with available literature correlations

There are several studies available which have investigated the responses of hot wires and films to changes in gas composition (see the discussion in §2.2). Most of

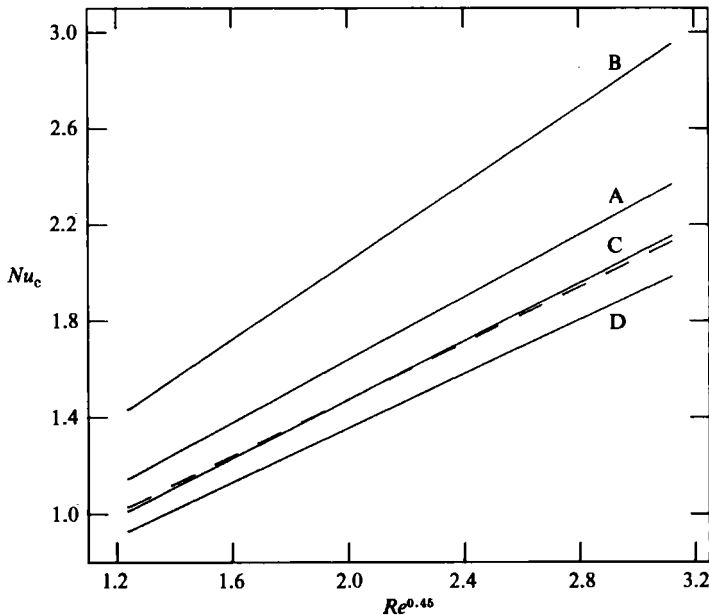


FIGURE 21. Correlations reported by Andrews *et al.* (1972) Collis & Williams (1959) and Fingerson & Ahmed (1976) are employed to give the solid lines (A, C and D respectively) shown as Nu_c versus $Re^{0.46}$ for the hot film placed in a flow of air. T_m is assumed to be 472 K and T_∞ to be room temperature. The correlation that has been obtained in this work for the hot film is also included on the plot (line B). The dashed line is the result which is found if the correlation of Andrews *et al.* (1972) is modified to reflect more recent literature measurements of air conductivity than that used by Andrews *et al.* (see the discussion in the text).

these studies have involved no more than three different gases and usually one of these was air.

The most common correlation suggested for the treatment of hot-wire and -film measurements in different gases is the combination of Collis & Williams' (1959) relationship for air with Kramers' (1946) results for heat transfer from cylinders placed in different fluids to give (19). This equation has been used or suggested by at least three groups of workers (Wu & Libby 1971; McQuaid & Wright 1973; Simpson & Wyatt 1973). Figure 12 shows that there is very little relationship between values of A_c or B_c for the various gases and the corresponding Pr . Perhaps more importantly, given the small variations in Pr among different gases, the exponents required on the Pr ratios in (19) would have to be many times larger than the values of 0.20 and 0.33 in order to reduce the data for the gases investigated here to a common curve. We conclude, based on these observations, that while it is possible that forced convective heat transfer from cylinders to gases may have a weak dependence on gas Pr , this dependence is hidden by the much stronger effect of molecular-property temperature variations on the heat transfer.

Fingerson & Ahmed (1976) are the only other workers of whom we are aware who have used a temperature-dependent property ratio to correlate heat-transfer results for anemometers placed in different gases. Their correlation for cooled films is given by (21). The ratio of ν_m/ν_∞ used in this correlation is similar to one of the corrections we have used to modify Nu_c (see (34)). As already noted, the exponents for the property ratios are very different for the two studies, but it is significant that both of these studies have required such a ratio to correlate heat-loss results in different

gases. Interestingly, these authors have reached the same conclusion as given above concerning the inability to discern an effect of Pr on the heat-transfer behaviour.

It is also of interest to compare our results with the conclusions of Andrews *et al.* (1972), who reported measurements of hot-wire response in air and mixtures of methane and nitrogen. These workers correlated their results for pure gases and mixtures in terms of Nu_a by correcting large-aspect-wire results for thermal-slip effects. The agreement was quite good, but in the light of the results given in this paper must be considered fortuitous. As figure 6 shows, values of Nu_c for methane and air plotted as functions of $Re^{0.45}$ do not lie on the same curve. The discrepancy in the findings of these two studies may be due to differences in the values of thermal conductivity used.

6.2. End conduction corrections

In §3.7 we have given formula for the correction of measured Nu made for finite-aspect-ratio probes. This correction compensates for heat losses to the prongs supporting the probe due to heat conduction along the probe. The resulting Nu corresponds to that for heat transfer from an infinite probe at the same average probe temperature. In making these calculations we have made several assumptions and approximations as discussed in §2.1. Additionally, a literature value of the thermal conductivity of tungsten was used for k_s even though the probe is known to have a layer of platinum deposited on its surface. Despite these approximations, we have obtained results for air which are in excellent agreement with measurements of hot-wire response where end conduction effects are known to be absent (Collis & Williams 1959; Andrews *et al.* 1972; Wu & Libby 1971). Barring the accidental cancellation of errors, this finding validates the procedure used to correct for heat losses by conduction along the hot wire.

It is extremely encouraging that the correction procedure leads to conclusions (i)–(v) listed in §4.1. These calculated effects of changes in heat-loss behaviour with aspect ratio are in complete agreement with experimental findings. This agreement lends further support to the model used here to make the end-loss corrections. The importance of making such corrections is clear from the finding that no correlation of our results in terms of Nu_m could be found.

In the case of the hot-film measurements, it is clear that our correction procedure only provides an approximate physical picture for end conduction heat losses. It is not surprising that our approximations fail in this case since heat transfer along the hot film is not expected to be one-dimensional. A more highly developed heat-transfer expression for conduction heat losses along the heated film needs to be developed in order to allow more exact correlations of our hot-film data in different gases.

6.3. Reynolds-number dependence of heat transfer

It has been shown that the hot-wire results for different gases can all be fit extremely well in terms of Nu_a using a value of $n = 0.45$ as the exponent for the Re . Table 4 lists the Re -ranges over which these measurements have been made. All of the measurements taken together cover an Re -range of approximately 0.1–6.2. Note that the results for some of the gases fall at the lower end of the range while those for others fall at the upper end. The fact that observed values of n_0 are so similar for all of the different gases requires that there be very little change in heat-transfer behaviour over this limited Re -range.

The situation for the hot film is very different. In this case, the Re -range covered is approximately 1.0–43. Gases which fall in the lower portion of this range (e.g. air, Ar and CH_4) have n_0 values of about 0.45. These values are consistent with those

found for the hot wire. On the other hand, the three gases for which results lie at the high end of the Re -range (C_3H_8 , SF_6 and CF_3Br) give n_0 values in the 0.36–0.39 range. Significantly, the measurements for CO_2 , which lie in an intermediate Re -range, give an n_0 value of 0.42 which is between the n_0 values found for the measurements made at the extremes of the Re -range. Note that all of the measurements are made for velocities low enough to ensure that the sharp flow transition at $Re \approx 44$ has not occurred.

The results of measurements made over the transition region mentioned in the last paragraph have been described in §4.2. Figures 9 and 10 show clearly that there is an abrupt transition in heat-transfer behaviour for this Re -range as described by Collis & Williams (1959) and much earlier by Hilpert (1933). Collis & Williams used hot-wire anemometry in the wake of the hot wire to show conclusively that the Re for which the transition occurs is identical with that for which eddy shedding commences from the heated probe.

The sharpness of this flow transition is demonstrated by the data plotted in figure 10. A clear hysteresis is observed in the heat-transfer behaviour depending on whether the velocity of the gas is increased or decreased to the measurement value. This observation implies that there is a narrow bistable Re -regime that lies between the flow velocities where eddies are not shed from the cylinder and those where such shedding always occurs. Closer inspection reveals that for the three cases investigated, the transition from no eddy shedding to the onset of shedding is extremely abrupt and occurs within a change in Re of about 0.6. The results for propane also show a sharp transition from flow with eddy shedding to flow without eddy shedding. Interestingly, the results for SF_6 and CF_3Br indicate that flows of these gases have a smooth transition in heat-transfer behaviour on going from the eddy-shedding to no eddy-shedding flow conditions. The differences in transition behaviour for SF_6 and CF_3Br compared with C_3H_8 result in the bistable region of the former two gases extending over a wider range of Re . At the present time we can offer no firm explanation for the differences in transition behaviour between C_3H_8 and the heavier molecular weight gases. It is intriguing to speculate that there may be a molecular-weight effect on this flow transition behaviour.

There are some additional points which should be made concerning the onset of vortex shedding. First, note that the Re -range covered by the transition regions is $36 \lesssim Re \lesssim 55$ which is nearly centred on the value of $Re \approx 44$ quoted by Collis & Williams (1959) as that for the onset of eddy-shedding flow behaviour. In their study of cooled films, Fingerson & Ahmed (1976) also report a flow transition between $Re = 40$ and 55. These three studies have been made for a wide range of temperatures varying from room temperature to 1700 K. The fact that the onset of eddy shedding is found at $Re \approx 44$ in all of these studies is a strong recommendation for the use of T_m in determining gas properties for use in the non-dimensional variables. If values of ν at different temperatures (e.g. T_s or T_∞) were used, the observed values of Re for the transition would vary widely. Minor variations are observed in the exact Re -ranges over which the hysteresis occurs for the different gases, as can be seen in figure 10. We attribute these to uncertainties in the values of ν used to calculate Re and to differences in the temperature dependencies of ν . In analogy with the discussion given in §6.1.2, such variations are expected when T_m is used to determine values of ν for use in calculating Re .

The behaviour of heat transfer from heated cylinders as a function of Re is related to the values of n_0 required to fit (2) for Nu_c (or Nu_a) as a function of Re^n . The findings of this study can be broken down into the following four Re -regimes:

I. $0.1 \lesssim Re \lesssim 5$: n_0 remains essentially constant at about 0.45 for measurements falling into this Re -range;

II. $5 \lesssim Re \lesssim 35$: n_0 decreases as the Re increases;

III. $35 \lesssim Re \lesssim 55$: This is the bistable region determined by the presence or absence of eddy shedding from the heated cylinder. Very low values of n_0 (< 0.35) are associated with gas flows for which no eddies are being shed. Large values of n_0 (> 0.50) result when eddies are present behind the cylinder; and

IV. $Re > 55$: Values of n_0 which fit the data in this Re -regime are ≥ 0.50 . We have not made extensive measurements in this Re -regime.

It is interesting to discover that Hilpert (1933) divided his heat-transfer results for heated cylinders in air into three regimes roughly corresponding to I, II and IV above with a break between II and IV at $Re \approx 40$. He fitted his experimental results to an equation of the form

$$Nu_\infty = C[Re(T_s/T_\infty)^{\frac{1}{2}}]^{n'}, \quad (37)$$

where values of C and n' appropriate for the given Re -ranges are listed in table 7. If Hilpert's results for Nu_∞ as a function of Re are fit to an equation of the form of (2) for $Re = 4$ to 40, n_0 is found to be 0.39, which is in good agreement with our experimental results for gases falling in this Re -range. Unfortunately, a similar calculation for the 1–4 Re -range gives a still smaller value of n_0 , which is the opposite of our experimental finding.

In general, Collis & Williams (1959) found good agreement with Hilpert's results, but argued that the transition at $Re \approx 4$ does not actually exist but is an artifact of the small number of experimental measurements used by Hilpert to deduce its existence. As shown by (3) and table 1, Collis & Williams only required two Re flow regimes to fit their experimental measurements which extended from $Re \approx 0.02$ to 140. A careful examination of the data given in their paper shows that the slow variation in n_0 that we find just before the flow transition at $Re \approx 44$ may not have been large enough to be discerned by these authors.

It is possible to speculate on the physical basis for a modification of heat-transfer behaviour on going from Re -regime I to II. This can be done by looking at the flow patterns which exist around an infinite cylinder in a gas flow (Whitaker 1968, 1977). At $Re < 4$ the flow past a cylinder does not display a wake and the flow moves over the cylinder as a non-separating flow. At $Re \approx 4$ the flow separates from the cylinder, a stagnation region develops behind the cylinder, and a wake appears. As Re is increased, the stagnation point moves slowly forward on the cylinder and recirculation eddies develop in the rear of the cylinder. These eddies grow and intensify until $Re \approx 44$ at which point alternate shedding of vortices occurs from the sides of the cylinder.

The effects of these transitions can be seen in a plot of the drag coefficient C_D as a function of Re (Schlichting 1960). The modification of C_D due to the change in flow behaviour at $Re \approx 4$ appears larger than that due to the transition at $Re \approx 44$ for the data plotted on a log-log plot.

It seems clear that the variations in n_0 observed over Re ranging from approximately 5 to 50 and for which vortex shedding is absent are due to the formation of stagnation points and the growth of recirculation zones behind the heated cylinder. For heat transfer from the cylinder this is a gradual and continuous change as reflected in the slowly decreasing values of n_0 as the Re -range of the measurements is increased. For $Re < 5$ the fluid flow corresponds to non-separated flow and no variation of n_0 with Re is expected. This conclusion is in agreement with experimental findings.

	$1 < Re < 4$	$4 < Re < 40$	$40 < Re < 400$
C	0.891	0.821	0.615
n'	0.330	0.385	0.466

TABLE 7. Values of C and n' reported by Hilpert (1933) for use in (37)

The above discussion indicates that the existence of Re regimes I, II and IV can be identified with the known flow behaviour of gases over a cylinder. Apparently, the hysteresis in heat-transfer behaviour, which is the distinguishing characteristic of regime III, has not been reported previously in the heat-transfer or hot-wire literature. However, such bistable regions are not uncommon in fluid mechanics and the existence of a hysteresis for vortex shedding from a cylinder should not be a surprise.

6.4. Accommodation effects

In §5 we argued that it is necessary to consider accommodation effects in correlating our hot-wire measurements in flows of helium and methane. For the remaining seven gases investigated α was assumed equal to 1. It seems advisable to question these assumptions as well as the plausibility of the values of α ($\alpha = 0.48$ for helium and $\alpha = 0.64$ for methane) determined.

Thermal accommodation behaviour is poorly understood, but is thought to arise on a molecular level owing to the incomplete transfer of heat from a heated surface to a molecule or atom striking the surface. The species colliding with the surface only 'accommodates' a fraction of the energy available from the surface with the result that the Boltzmann temperature of the gas-phase species near the surface is lower than expected based on the surface temperature. Theories which treat this effect are usually based on the temperature jump which exists at the boundary of the solid and gas. The result of this temperature jump is that heat transfer is reduced from the value that would be found if it did not exist.

The primary impression that results from a review of the literature on thermal accommodation is the degree of complexity of the effect. In particular, a great deal of variability in measured values of accommodation coefficients is observed unless extremely clean and well-characterized surfaces are employed (Thomas 1967). This variability has severely limited attempts to understand and predict values of α . The effects of such important system parameters as surface temperature, gas pressure, and gas molecular weight on thermal accommodation are poorly characterized and understood.

Despite the confusing and sometimes contradictory experimental findings concerning the characteristics of thermal accommodation, the following general observations are believed to be correct (Deviene 1967). The degree of surface 'cleanness' is critical to measured values of α . Cleaner surfaces result in lower values of α . Values of α for polycrystalline or ordinary industrial matrices 'which are macroscopically clean but contaminated with miscellaneous physically and chemically absorbed materials' (Flood & Hobson 1967) are expected to be substantially larger than those found for carefully prepared surfaces. Increasing molecular weight and complexity are associated with increasing values of α . There is an apparent effect of surface temperature on the degree of thermal accommodation, but the experimental results are contradictory and no definitive trends are discernible (Deviene 1967). The effect

of pressure on observed values of α is believed to be small except for cases where the pressure determines whether or not gas layers are present on the surface.

The extreme sensitivity of thermal accommodation to so many system parameters makes prediction of values of α nearly impossible except for experiments made under highly controlled conditions. Measurements with hot wires and films certainly do not fall into this class. The surface structures of these probes are not well characterized and are expected to contain very uneven and variable surfaces due to the manufacturing processes used in their construction. Furthermore, these probes are usually operated in environments that are expected to continuously contaminate and modify the probe surface. In this study involving the use of several different gases, the nature of gases absorbed on the surface of the probe might also be expected to vary from experiment to experiment.

The above discussion makes it clear that one cannot expect to theoretically predict values of α for use in hot-wire anemometry. The best one can hope to do is determine values of α phenomenologically and compare the experimental results with the general characteristics of thermal accommodation.

In the case of the work reported here, the first assumption that should be questioned is the use of $\alpha = 1$ for all of the gas flows over the hot wire and film with the exception of the wire results for helium and methane. Andrews *et al.* (1972) have listed selected values of α for a wide range of surfaces and gases. The variability of results found in measurements of α is quite clear from this table. Despite this it is possible, by using the reported values of α along with the expected modifications due to the hot-wire-probe surface conditions, to estimate that the value of α will lie between 0.9 and 1 for air on the hot wire. Based on the molecular and complexity arguments described above, the same estimate will apply to all of the higher-molecular-weight gases investigated (i.e. all except helium and methane). Values of α should be very nearly equal to one for all of these gases on the alumina surface of the hot film.

These estimates of α can be used to evaluate the effect of thermal accommodation on the correlations of our hot-wire and -film results. α enters our calculations of Nu_a in (8) as a factor in the thermal-slip parameter Δ . In calculating Δ the value of what we have referred to as the 'classical' slip parameter is multiplied by $(2-\alpha)/\alpha$. For an α -value of 0.9 this corresponds to an increase of 22% in the value of Δ over that expected in the absence of thermal-accommodation effects. Since values of Δ enter directly into (11) the parameter ϕ will also increase by 22%. ϕ is used in (10) to correct Nu_a for non-continuum effects. For air the value of ϕ is calculated to be 1.85 if $\alpha = 1$. By substituting this value of ϕ into (10) along with the Kn for air one finds that a value of $Nu_c = 1.050$ is calculated if $Nu_\infty = 1.00$ is assumed. The corresponding value of ϕ is 2.260 for $\alpha = 0.9$, which gives a value of $Nu_a = 1.062$ when Nu_∞ is corrected for non-continuum effects. The value of Nu_a is only 1% larger than Nu_c . This result shows that the use of $\alpha = 1$ gives results for Nu_c which differ only slightly from the values of Nu_a that would be calculated if the actual values of α for air were known. This conclusion will be true for the hot-wire measurements made in air, Ar, CO_2 , C_3H_8 , CF_4 , SF_6 and CF_3Br . It justifies our conclusion that it is possible to correlate the results for these gases (see figures 16–18) in terms of Nu_c . The errors introduced in the correlation by the use of Nu_c instead of Nu_a are considerably smaller than those arising from other sources such as uncertainty in molecular-property values. This approximation will be even more correct for the hot film, for which values of α should be even closer to one than for the hot wire.

The effect of accommodation on hot-wire-anemometry measurements has been investigated primarily for helium. Libby and coworkers were the first researchers to note the importance of the small accommodation coefficients for this gas on the metal surfaces of hot-wire probes. For tungsten wires, values of $\alpha = 0.030$ (Aihara *et al.* 1967) and $\alpha = 0.040$ (Baccaglini *et al.* 1969) were reported. A later study (Wu & Libby 1971) of the heat transfer from a platinum hot wire gave $\alpha = 0.11$. In all three cases the values of α were shown to fall within the ranges reported in literature measurements of helium on the appropriate surfaces. For the results on tungsten, Baccaglini *et al.* (1969) indicate that researchers in the area of gas-surface interactions had questioned the very small values of α determined because of the contaminated nature of the hot-wire surfaces. These workers went on to state that they 'have sufficient confidence in our technique to take the view that reasons for the low values of α require further study by researchers concerned with gas-surface interactions'.

Our experimental determination of α for helium on the platinum surface of the hot wire gave $\alpha = 0.48$. We feel that this result is an accurate estimate of α not only because it yields values of Nu_a for helium which are well correlated with our results for seven other gases, but also because it gives a value of n_0 for data plotted as Nu_a versus Re^n which is in excellent agreement with those found for measurements in other gases. The value of n_0 is highly dependent on the magnitude of α as it changes from 0.35 for $\alpha = 1$ to 0.45 for $\alpha = 0.48$.

Our value of α for helium on the platinum hot-wire surface is consistent with values found in the literature. Devienne (1967) lists an average value of $\alpha = 0.54$. However, as noted above, the reader is cautioned that the experimental results quoted by Devienne covered an extensive range of values.

It is difficult to suggest a reason why our value of α for helium is so much larger than those found by Libby and coworkers (Aihara *et al.* 1967; Baccaglini *et al.* 1969; Wu & Libby 1971). The most obvious possibility is that the surface properties of the probes are sufficiently different to account for the variations. Even though the earlier works were not corrected for the temperature variations of fluid properties, these variations are so similar for helium and air that they can account for only a small fraction of the differences observed.

Our results indicate that there is also a small accommodation effect ($\alpha = 0.61$) for methane. This is reasonable since methane is lighter than air and has high-energy vibrational modes. For these reasons, this molecule is expected to have a value of α intermediate between that of helium and air, as is observed.

One other study has reported accommodation effects on hot-wire measurements. Brown & Rebollo (1972) considered such effects in their development of a concentration probe which incorporated a hot-wire. These workers indicate that it is necessary to account for accommodation effects for Ar on platinum in correlating their experimental results. This disagrees with our findings. However, these workers did not correct their measurements for end heat-loss effects or the temperature dependence of gas properties. The neglect of such effects can lead to erroneous conclusions concerning the importance of accommodation effects on heat-transfer behaviour.

Based on the above discussion, we feel that, while neither justified by past practice or a strong theoretical basis, our procedures for treating accommodation effects are sufficiently accurate for correlation purposes. Our general findings concerning this effect are in good agreement with the trends expected (e.g. with respect to the effect of molecular weight).

6.5. *Some comments on hot-wire and -film calibrations*

There are a number of opinions in the literature concerning procedures to be used in calibrating the response of hot-wire anemometers (Bradshaw 1971). Disagreements concern such general questions as whether universal correlation laws should be used at all and more specific questions such as which exponent to use in fitting calibration data to equations of the form of (26) or (2). While we do not wish to join this continuing controversy, a few comments based on the findings of this work are relevant. Perhaps the major finding is that hot-wire and -film responses to flows of different gases can be correlated. If accommodation effects are expected to be absent and molecular-property data are available, it should be possible to predict the response of the probe in a second gas following calibration in a first gas. This finding could provide a considerable saving of time and effort for experiments in which the flow velocity of several gases are to be investigated. In generating these correlations it is necessary to correct the measurements for end-heat-loss effects, temperature variation of properties, and 'classical' non-continuum effects. The question of whether to use an equation of the form of (2) often arises in calibrating hot wires. Variations of the value of n_0 with Re are often discussed (Hinze 1975). Our work indicates that these variations are due primarily to changes in the flow behaviour with Re and to non-continuum effects for small probes and light-molecular-weight gases. The choice of an equation to fit experimental calibration data should be based on the Re at which the probe will be operated. Our findings indicate that either (2) or (26) will be appropriate if the probe Re falls in the 0.5–6 range. The best value of n to use will depend on exactly how the data is treated. For instance, all of our hot-wire data, with the exception of helium, can be fitted quite accurately by assuming a value of $n = 0.43$ and using either (26) or (2) in terms of Nu_m . Note that these forms will not allow a correlation of data for different gases. On the other hand a value of $n = 0.45$ is more suitable for data treated as Nu_c or Nu_a . In making such assessments, care must be taken to identify non-continuum flow effects. If measurements are made in the Re -regime from 6 to 40 the use of (2) becomes questionable as we have shown that n_0 values vary over this range. Measurements in the bistable range of Re from approximately 35 to 55 should be avoided if at all possible.

6.6. *Heat-transfer studies using hot wires or films*

The hot wires and films investigated here are employed primarily as velocity measurement devices and this study is designed to investigate this use. However, it is worthwhile to point out that these devices are quite sensitive to changes in convective heat transfer and are therefore able to provide quite detailed measurements of heat transfer from cylinders.

This study has generated information on the behaviour of heat transfer in the Re range from 0.5 to 55. The results have allowed us to reach conclusions concerning the effects of flow transitions on heat-transfer behaviour. A very clear hysteresis in heat-loss behaviour has been observed and associated with the presence or absence of eddy shedding from the cylinder. Use of a large number of different gases have allowed us to identify the importance of fluid-property temperature dependence when the non-dimensional parameters are calculated in terms of gas properties evaluated at T_m . All of these effects were poorly characterized in the literature. Clearly, our investigation of heated velocity probes has generated a great deal of new information of interest to workers in the area of heat transfer.

It is also of interest to note that with further development measurements of the type reported here might yield values of gas thermal conductivity or viscosity. For instance, if the types of correlations that we have used here are shown to be reproducible and universal, it would be relatively simple to determine a value of k for a gas if the corresponding value of ν were available. Since accurate values of ν are easier to measure experimentally, it will often be true that they will be available when k -values are not. Measurements of this type should be possible that have a precision, and perhaps accuracy, of about 1%.

Measurements of this type also seem to offer the promise of a method for measuring accommodation coefficients for gases on surfaces in a relatively straightforward and rapid manner. It may be possible to develop a technique based on hot-wire anemometry for studying this complex and poorly understood phenomenon.

7. Final remarks

This paper has described hot-wire and -film calibrations that have been made for a wide variety of gases. The use of gases having a broad range of gas molecular properties has allowed us to isolate the corrections which must be made in order to correlate the data in the usual form of Nu as a function of flow Re . These corrections include end heat losses due to conduction along the probe, a very strong effect due to the temperature dependence of gas molecular properties, 'classical' non-continuum effects, and accommodation effects on heat transfer from the probe surfaces to the gases. All of these effects are known and each has been used separately in the literature in attempts to correlate hot-wire or film responses. This is the only study of which we are aware that has investigated the effects of each correction on heat-transfer behaviour and has been able to develop a systematic procedure that allows the correlation of such a wide range of experimental data.

In the case of the hot wire the resulting Nu (Nu_c or Nu_a depending on whether or not accommodation effects are important) has a dependence on Re in air which is very close to that found for previous hot-wire measurements where end heat losses are absent and non-continuum effects have been accounted for. This observation offers the possibility that universal calibration laws may be possible to describe the response of a hot wire. The agreement for the hot-film results is not nearly as complete. We have attributed this to the approximation of a one-dimensional heat loss used to treat the heat losses at the end of the probe. It should be possible to develop better models to treat this process.

The finding that the heat-loss behaviour of these devices (as reflected in the value of n_0 for fitting data to the form of (2)) is strongly dependent on the nature of the flow around the probe should also allow a much better prediction of probe response for a given Re -range. These results should be considered when choosing a probe for a particular application.

Only one overheat ratio has been used for the hot wire. A slightly larger value has been used for the hot film. For this reason, this study offers no conclusions regarding the effect of overheat ratio on heat-transfer behaviour. It is impossible to conclude whether the probe temperature dependence noted in past experiments is due to property temperature dependencies or to other possible effects such as buoyancy, etc. The role of offset-voltage setting on constant-temperature hot-wire anemometry may also be important. Further studies should be done to clarify these points.

Our major goal for this work was to be able to understand and perhaps predict the response of a heated cylindrical probe to convective flows of different gases. The

experiments reported here have discussed our findings for pure gases. For these gases great strides have been made toward our goal. A second paper will describe the results for mixtures of different gases.

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